ANALYSIS AND OPERATORS: SOME ASPECTS OF RECENT PROGRESS AT EUROPEAN LEVEL, 2000-2004

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The European network "Analysis and operators", which operated from June 1, 2000 to May 31, 2004, had ten nodes located at Bordeaux, Amsterdam, Barcelona, Dublin, Leeds, Paris 6, Trondheim, Vienna, Tel-Aviv and Saint Petersburg. The three main directions of research were Function Theory, Operator theory, Geometry of Banach spaces and Convex Geometry. The interplay between these branches of mathematics was structured in the network workplan along six tasks;

Hardy and Bergman spaces, Interpolation - Cauchy Integral, Capacities, Harmonic approximation- Function models and applications of operator theory-Invariant subspaces - Geometry of Banach spaces and applications-Convex Geometry, concentration of measures.

During the four years of network activity there were a number of individual breakthroughs (see for example the talks by Tolsa, who also participates to the network HARP, and Sodin at 4ecm), but we will concentrate in the present talk on joint work by members of different teams. We selected among many joint papers the following works

Characterization of Fourier frames of Duffin and Schaeffer in terms of de Branges' spaces (Barcelona-Trondheim)

Approximation on the boundary and sets of determination for harmonic functions (Barcelona - Dublin)

Noncyclic functions without zeroes and product of Toeplitz operators on weighted Bergman spaces (Bordeaux-Paris-Trondheim and Bordeaux-Leeds)

Schur complement of some self-adjoint operator matrices (Vienna-Tel-Aviv), exact computation of the Schur multiplier norm of Toeplitz matrices (Bordeaux-Saint Petersburg)

Application of the smooth variational principle to the existence of invariant subspaces (Paris-Tel-Aviv), and applications of invariant subspaces of infinite hamiltonians to Riccatti equations (Amsterdam-Vienna).

There were no major difficulties to appoint pre or post-docs, even if the number of applicants was not always very large, and on several occasions the post-docs produced as expected joint papers with a senior member of the host team. Network activity could not multiply the number of female participants, but it certainly helped their mathematical activity and a third of the authors of the papers mentioned above are women.

We will conclude by presenting two concrete experiences of collaborations between analysts and engineers: in the Amsterdam node it took place with a department of chemical engineering and in the Bordeaux node with a laboratory of automated systems and signal processing. To develop at European level such an interaction between powerful abstract mathematical methods ranging from analysis to operator theory and engineers' know-how will be one of the main challenges for future networks in these mathematical areas.