HYDRODYNAMIC LIMITS

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Hydrodynamic limits are the mathematical process by which the models governing the evolution of a continuous medium are derived from the laws of motion of its microscopic constituents. For instance, the classical PDEs of gas dynamics (i.e. the Euler or Navier-Stokes equations) can be formally deduced from the system of ODEs obtained by applying Newton's second law of motion to each gas molecule [C.B. Morrey, Comm. on Pure and Appl. Math. 8 (1955)]. However, obtaining a complete mathematical justification of these formal arguments seems exceedingly difficult at present. Indeed, this would most likely require a better understanding of the ergodic theory of infinite particle systems, and of the regularity theory for the nonlinear PDEs of Fluid Mechanics (such as the Navier-Stokes equations).

The purpose of this lecture is to describe some recent progress (in collaboration with L. Saint-Raymond) on one aspect of this question, namely the derivation of the Navier-Stokes equations from the Boltzmann equation in the kinetic theory of gases. In the classical methods of proving such hydrodynamic limits pioneered by D. Hilbert [Math. Ann. 72, (1912)], the regularity of solutions to the target equation played an essential role.

To avoid the limitations in these methods due to the possible apparition of singularities in finite time in either the Boltzmann or the Navier-Stokes equations, it was proposed in [C. Bardos, F. Golse, C.D. Levermore, Comm. on Pure Appl. Math. 46 (1993)] to work with the global weak solutions constructed by J. Leray [Acta Math. 63 (1934)] for the Navier-Stokes equations, and by R.J. DiPerna and P.-L. Lions [Ann. of Math. 130 (1993)] for the Boltzmann equation. This approach requires new mathematical tools, intrinsically adapted to hydrodynamic limits, and unaffected by the possible loss of regularity in the solutions considered.