

CONSTRUCTING ALGEBRAIC VARIETIES VIA COMMUTATIVE ALGEBRA

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Problems on the existence and moduli of abstract varieties in the classification of varieties can often be studied by embedding the variety X into projective space, preferably in terms of an intrinsically determined ample line bundle L such as the (anti-)canonical class or its submultiples. A comparatively modern twist on this old story is to study the graded coordinate ring

$$R(X, L) = \bigoplus_{n \geq 0} H^0(X, L^n),$$

which in interesting cases is a Gorenstein ring; this makes available theoretical and computational tools from commutative algebra and computer algebra. The varieties of interest are curves, surfaces, 3-folds, and historical results of Enriques, Fano and others are sometimes available to serve as a guide. This has been a prominent area of work within European algebraic geometry in recent decades, and the lecture will present the current state of knowledge, together with some recent examples.