THE GL_2 MAIN CONJECTURE FOR ELLIPTIC CURVES WITHOUT COMPLEX MULTIPLICATION

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The complex Hasse-Weil L-function $L(E/k, s)$ of an elliptic curve E over a number field k encodes properties of the reduction of E modulo every prime ideal. Though locally in nature it contains conjecturally a large amount of global information of the curve. More precisely, the Birch and Swinnerton-Dyer Conjecture predicts that $L(E/k, S)$ has a zero at $s = 1$ whose order equals the rank of the Mordell-Weil group $E(k)$ and it relates the corresponding leading coefficient to arithmetic invariants of E. Following Iwasawa's philosophy it is often more effective to study the whole family of the values at $s = 1$ of the L-function twisted by Artin characters all at once, which leads to the question whether there exists a p-adic analytic L-function which interpolates these special values. On the arithmetic, i.e. Galois cohomology side this corresponds to considering t he Pontryagin dual of the Selmer group of E over an p-adic Lie-extension of k with Galois Group G as module under the Iwasawa algebra of G thereby hopefully obtaining a characteristic element attached to E in purely algebraic terms. The Main Conjecture would then state that the p-adic L-function and the characteristic element coincide essentially. While for elliptic curves with complex multiplication this program has almost been completed by the work of Rubin and others, in the GL_2-situation even the precise formulation forms a serious problem. In this talk, after a short historical retrospection, we discuss the approach to characteristic elements which has recently been initiated by the speaker and extended to the general case in a joint work with J. Coates, T. Fukaya, K. Kato and R. Sujatha. Assuming the existence of p-adic L-functions we conclude stating the Main Conjecture in the GL_2-case.