SCHRAMM-LOEWNER EVOLUTIONS, BROWNIAN LOOP-SOUPS AND CONFORMAL RESTRICTION

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Systems from statistical physics taken at their critical temperature can exhibit random behaviour at macroscopic scale. It had been recognized by theoretical physicists more than twenty years ago that conformal invariance should play an important role in the understanding of this random behaviour in the case of two-dimensional systems. This has led among other things to the development of conformal field theory and to various striking predictions (Belavin-Polyakov-Zamolodchikov, Cardy, Nienhuis, Duplantier, Saleur...). As Oded Schramm will have explained in his lecture, there has been some recent mathematical progress in the mathematicians' understanding of these phenomena. In particular, the conjectural scaling limits of critical two-dimensional interfaces (now called Schramm-Loewner Evolutions – SLE) can be constructed via iterations of random conformal maps.

In the present lecture, I shall focus on two aspects: The conformal restriction property of a random subset K of a given set D that can be viewed as the invariance of its law under the group of conformal maps. All such random sets can be described and it turns out they are all closely related to each other. This allows for instance to identify the outer boundary of a planar Brownian loop, that of the scaling limit of a large percolation cluster, and the conjectural scaling limit of self-avoiding polygons (joint work with Greg Lawler and Oded Schramm). I will then describe the Brownian loop-soups (defined in a joint paper with Greg Lawler), which are random Poissonian collections of Brownian loops in a planar domain, with a conformal restriction-type property. In many cases, its complement defines a random Sierpinsky-carpet-like fractal set, with built-in conformal invariance properties. It turns out that these rather simple objects are closely related to SLE, to conformal field theory, and to some highest-weight representations of the Virasoro Algebra.