BRÅKET

Information om seminarier och högre undervisning i matematiska ämnen i Stockholmsområdet

NR 35 **FREDAGEN DEN 3 NOVEMBER 2000**

BRÅKET

Veckobladet från Institutionen för matematik vid Kungl Tekniska Högskolan och Matematiska institutionen vid Stockholms universitet

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Sista manustid för nästa nummer:
Torsdagen den 9 november kl. 13.00.

Kurs

Dževad Belkić: Resolution Enhancement in Signal and Image Reconstructions / continued (Master Equations in MRI and CT). Se sidorna 11 – 19.

Money, jobs: Se sidorna 9 – 10.

SEMINARIER

Må 11 – 06 **kl. 13.15 – 15.00. Algebra and Geometry Seminar. Gautami Bhowmik,** Lille: *Algebra of arithmetical functions of matrices.* Rum 306, hus 6, Matematiska institutionen, SU, Kräftriket, Roslagsvägen 101. Se sidan 4.

Må 11 – 06 **kl. 15.00 – 16.00. Prisutdelning och seminarium. Simon Singh, m.fl.:** *Att bevara sina hemligheter och avslöja andras.* Sal Q1, KTH, Osquldas väg 4, b.v. Se sidan 5.

Må 11 – 06 **kl. 15.15 – 17.00. Seminarium i matematisk statistik. Torkel Erhardsson:** *Compound Poisson approximation for visits to rare sets by certain stationary Markov chains and renewal reward processes.* Seminarierum 3733, Institutionen för matematik, KTH, Lindstedtsvägen 25, plan 7. Se Bråket nr 34 sidan 5.

Ti 11 – 07 **kl. 10.15. Plurikomplexa seminariet. Timur Sadykov,** SU: *Singularities of nonconfluent hypergeometric functions in several variables.* Sal MIC 2215, Matematiska institutionen, Polacksbacken, Uppsala universitet. Se sidan 4.

Ti 11 – 07 **kl. 13.15. Seminar in Theoretical Physics. Rikard von Unge,** Brno: *S-duality of noncommutative gauge theory and noncommutative open string theory.* Rum 4731, Fysikum, SU, Vanadisvägen 9. Se sidan 7.

Ti 11 – 07 **kl. 13.30. Plurikomplexa seminariet. Frank Kutzschebauch,** Uppsala: *Oka's principle with group action.* Sal MIC 2215, Matematiska institutionen, Polacksbacken, Uppsala universitet. Se sidan 4.

Fortsättning på nästa sida.

Prisutdelning

Ett svenskt lag har vunnit tävlingen *The Cipher Challenge.* Prisutdelningen äger rum den 6 november. Se sidan 5.

Seminarier (fortsättning)

Ti 11–07 kl. 14.00 – 15.00. Mittag-Leffler Seminar. Pauli Väisänen, Helsingfors: *Long games in almost free abelian groups.* Institut Mittag-Leffler, Auravägen 17, Djursholm.

Ti 11–07 kl. 15.00 – 17.00. Artinian Gorenstein rings and Frobenius algebras. Joachim Kock: *Introduction to formal Frobenius manifolds.* Sammanträdesrum 3548, Institutionen för matematik, KTH, Lindstedtsvägen 25, plan 5.

On 11–08 kl. 10.15. Kombinatorikseminarium. Nikolai Mnev, Steklov Institute, St. Petersburg: *About the category of abstract triangulations of a manifold.* Seminarierum 3733, Institutionen för matematik, KTH, Lindstedtsvägen 25, plan 7. Se sidan 6.

On 11–08 kl. 13.15. Seminarium i analys och dynamiska system. Peter Ebenfelt: *On mapping problems in CR geometry.* Seminarierum 3721, Institutionen för matematik, KTH, Lindstedtsvägen 25, plan 7. Se sidan 5.

On 11–08 kl. 14.00 – 15.00. Mittag-Leffler Seminar. Paul Eklof, Irvine: *Tutorial: Combinatorics of abelian groups, part II.* Institut Mittag-Leffler, Auravägen 17, Djursholm.

On 11–08 kl. 15.15 – 16.00. Seminarium i matematik och fysik vid Mälardalens högskola (Västerås). Hillevi Gavel, Institutionen för matematik och fysik, Mälardalens högskola: *Permutationsordningar och relationsmatriser.* Rum N24, Mälardalens högskola, Högskoleplan, Västerås. Se Bråket nr 34 sidan 5. Internetadressen till information om seminariet är http://www.ima.mdh.se/_seminars.htm.

On 11–08 kl. 15.30 – 16.30. Mittag-Leffler Seminar. Oleg Belegradek, Istanbul: *Polyregular ordered abelian groups.* Institut Mittag-Leffler, Auravägen 17, Djursholm.

To 11–09 kl. 13.15 – 14.15. Dynamiskt systemseminarium. Professor David Broomhead, Manchester: *Delay methods applied to iterated function systems.* Seminarierum 3721, Institutionen för matematik, KTH, Lindstedtsvägen 25, plan 7. Se Bråket nr 34 sidan 6.

To 11–09 kl. 13.30 – 15.00. Seminarium i statistisk genetik och bioinformatik. Stefan Arnborg, NADA, KTH: *Human Brain Informatics: understanding causes of mental illness.* Sal 21, hus 5, Matematiska institutionen, SU, Kräftriket, Roslagsvägen 101. Se Bråket nr 34 sidan 6.

To 11–09 kl. 15.15. Algebraic Geometry Seminar. Tom Graber, Harvard University, USA: *Hurwitz numbers and moduli of curves.* Sammanträdesrum 3548, Institutionen för matematik, KTH, Lindstedtsvägen 25, plan 5. Se sidan 7.

Fr 11–10 kl. 9.00 – 10.00. Kollokvium i fysik. Professor Gabor A. Somorjai, Department of Chemistry, UC Berkeley: *TBA.* Sal F01, Fysiska institutionen, KTH, Lindstedtsvägen 24, b.v.

Fr 11–10 kl. 15.15. Doktorandseminarium. Johan Andersson: *Riemanns zeta-funktion och Poissons summationsformel för* $SL(2, Z)$. Rum 306, hus 6, Matematiska institutionen, SU, Kräftriket, Roslagsvägen 101. Se sidan 7.

Fortsättning på nästa sida.

Seminarier (fortsättning)

Må 11–13 kl. 11.15 – 12.00. Matematikens år 2000. Professor John Conway, Princeton: *FRACTRAN — a logical programming language.* Sal E1, KTH, Lindstedtsvägen 3, b.v. Se sidan 6.

Professor Conways föredrag anordnas av Kungl. Vetenskapsakademien och Institutionen för numerisk analys och datalogi (NADA), KTH.

To 11–16 kl. 10.30 – 11.15. Waveletseminarium. *(Observera lokalen!)* **Professor Dževad Belkić,** Medicinsk strålningsfysik, Karolinska institutet och Stockholms universitet: *Noise reduction in generic signals, spectra and images.* Rum 1537, NADA, KTH, Lindstedtsvägen 3, plan 5. Se sidan 8.

To 11–16 kl. 11.00. Seminarium i statistisk genetik och bioinformatik. Juha Kere, Genome Centre, Helsingfors Universitet: *Finding genes in multifactorial diseases — a rare population approach.* Sal 14 (Gradängsalen), hus 5, Matematiska institutionen, SU, Kräftriket, Roslagsvägen 101. Se Bråket nr 34 sidan 6.

To 11–16 kl. 11.50. Seminarium i statistisk genetik och bioinformatik. Holger Luthman, Clinical Genetics, Karolinska institutet: *Animal models for multifactorial diseases.* Sal 14 (Gradängsalen), hus 5, Matematiska institutionen, SU, Kräftriket, Roslagsvägen 101. Se Bråket nr 34 sidan 6.

To 11–16 kl. 13.30. Seminarium i statistisk genetik och bioinformatik. Joe Terwilliger, Columbia University, New York: *Title to be announced.* Sal 14 (Gradängsalen), hus 5, Matematiska institutionen, SU, Kräftriket, Roslagsvägen 101. Se Bråket nr 34 sidan 6.

To 11–16 kl. 14.20. Seminarium i statistisk genetik och bioinformatik. Paul Burton, Genetic Epidemiology, Leicester University, UK: *Generalized linear mixed models in statistical genetics.* Sal 14 (Gradängsalen), hus 5, Matematiska institutionen, SU, Kräftriket, Roslagsvägen 101. Se Bråket nr 34 sidan 6.

To 11–16 kl. 15.30. Seminarium i statistisk genetik och bioinformatik. David Clayton, Cambridge University, UK: *Title to be announced.* Sal 14 (Gradängsalen), hus 5, Matematiska institutionen, SU, Kräftriket, Roslagsvägen 101. Se Bråket nr 34 sidan 6.

To 11–16 kl. 15.15 – 16.00. Seminarium i matematik och fysik vid Mälardalens högskola (Eskilstuna). Torgil Abrahamsson, Institutionen för matematik och fysik, Mälardalens högskola: *Estimation of Origin-Destination (OD) matrices using traffic counts.* Rum B315, Mälardalens högskola, Eskilstuna. Internet-adressen till information om seminariet är http://www.ima.mdh.se/_seminars.htm.

Fr 11–17 kl. 9.00 – 10.00. Kollokvium i fysik. Professor Laszlo Kish, Ångströmlaboratoriet, Uppsala: *Peculiar effects in nanoparticle tungsten oxide and gold films with potential technological applications.* Sal F01, Fysiska institutionen, KTH, Lindstedtsvägen 24, b.v.

ALGEBRA AND GEOMETRY SEMINAR

Gautami Bhowmik: Algebra of arithmetical functions of matrices

Abstract: We establish a bijection between divisor classes of integer matrices and lattices. This gives an isomorphism between the algebra of arithmetical functions of matrices and an extension of the Hecke algebra. We introduce zeta functions on this algebra and get interesting results on abelian groups and Hall polynomials.

Tid och plats: Måndagen den 6 november kl. 13.15 – 15.00 i rum 306, hus 6, Matematiska institutionen, SU, Kräftriket, Roslagsvägen 101.

PLURIKOMPLEXA SEMINARIET

Timur Sadykov: Singularities of nonconfluent hypergeometric functions in several variables

Abstract: I will present a joint work with Mikael Passare and August Tsikh. Typically a nonconfluent hypergeometric function is a multi-valued analytic function with singularities along an algebraic hypersurface. The purpose of this talk is to give a characterization of the hypersurfaces which arise as singularities of multivariate hypergeometric functions. We prove that any meromorphic nonconfluent hypergeometric function is rational, and identify a class of rational hypergeometric functions with the Bergman kernels of complex ellipsoidal domains.

Tid och plats: Tisdagen den 7 november kl. 10.15 i sal MIC 2215, Matematiska institutionen, Polacksbacken, Uppsala universitet.

PLURIKOMPLEXA SEMINARIET

Frank Kutzschebauch: Oka's principle with group action

Abstract: The "Oka-priciple" can be vaguely stated as follows (see GRAUERT & REMMERT, *Stein spaces*):

> *On a reduced Stein space X, problems which can be cohomologically formulated have only topological obstructions. In other words, such problems are holomorphically solvable if and only if they are continuously solvable.*

We will present a theorem which fits into this philosophy. The theorem involves a holomorphic action of a compact, respectively a complex reductive, group. Also we will give some applications of the theorem to automorphisms of $\mathbb{C}^n$. Those are the up to now strongest positive results about linearization of holomorphic actions of compact (e.g. finite) groups on $\mathbb{C}^n$.

Tid och plats: Tisdagen den 7 november kl. 13.30 i sal MIC 2215, Matematiska institutionen, Polacksbacken, Uppsala universitet.

PRISUTDELNING OCH SEMINARIUM

Simon Singh samt
Fredrik Almgren, Gunnar Andersson, Torbjörn Granlund,
Lars Ivansson och Staffan Ulfberg:
Att bevara sina hemligheter och avslöja andras

Simon Singh berättar i *Kodboken* om kryptografins historia, som ofta varit en spännande kapplöpning, ibland bokstavligen på liv och död, mellan konstruktörer av krypteringssystem och forcörer som sökt efter svagheter i systemen för att kunna avslöja hemliga meddelanden. "Kodboken" avslutas också med en kapplöpning, *The Cipher Challenge,* en utmaning i tio steg som går ut på att avslöja tio hemliga meddelanden. Stegen speglar kryptografins historia. De första meddelandena är krypterade med enkla metoder från före Kristi födelse. Svårighetsgraden ökar sedan successivt, och via system som använts i de båda världskrigen når man tävlingens tionde och mest svårforcerade meddelande som krypterats med ett modernt system som bland annat används av det svenska bankväsendet.

För en knapp månad sedan avgjordes tävlingen då ett svenskt lag blev först med att knäcka det tionde meddelandet och därmed tog hem segern. Priset, 10 000 pund, delas ut måndagen den 6 november, och i samband med det kommer Simon Singh och det vinnande laget att ge ett seminarium om kryptografins historia och teknik i allmänhet, och om arbetet med "The Cipher Challenge" i synnerhet.

Simon Singh doktorerade i fysik vid University of Cambridge och har gjort sig känd som en skicklig populärvetenskaplig journalist och författare. I Sverige är han mest känd för sina böcker *Fermats gåta* och *Kodboken.*

Det vinnande laget är *Fredrik Almgren* (Smarttrust), *Gunnar Andersson* (Prover Technology), *Torbjörn Granlund* (SWOX AB), *Lars Ivansson* (Stockholm Bioinformatics Center) och *Staffan Ulfberg* (Quadriga Software).

Tid och plats: Måndagen den 6 november kl. 15.00 – 16.00 i sal Q1, KTH, Osquldas väg 4, b.v.

SEMINARIUM I ANALYS OCH DYNAMISKA SYSTEM

Peter Ebenfelt:
On mapping problems in CR geometry

Abstract: Since every real-analytic curve M in the complex plane is locally equivalent to the real line, the set of, say a priori continuous, boundary values on M of holomorphic functions sending M into another real-analytic curve M' is well understood but not very exciting. In higher-dimensional complex spaces, the situation is more interesting due to the fact that real submanifolds inherit a partial complex structure (a CR structure) from the ambient space. In this talk, we shall survey some recent directions in this field. The talk is aimed at a broad audience, and no special knowledge of complex analysis in several variables or CR geometry will be assumed.

Tid och plats: Onsdagen den 8 november kl. 13.15 i seminarierum 3721, Institutionen för matematik, KTH, Lindstedtsvägen 25, plan 7.

KOMBINATORIKSEMINARIUM

Nikolai Mnev:
About the category of abstract triangulations of a manifold

Abstract: We study the category $\mathbf{CM}(X)$ of all combinatorial manifold structures on a given compact PL-manifold X. Objects of $\mathbf{CM}(X)$ are abstract simplicial complexes S whose geometric realizations are PL-homeomorphic to X. Morphisms are "combinatorial subdivisions". Our result is a homotopy equivalence

$$B\mathbf{CM}(X) \approx B\mathrm{PL}(X),$$

where $B\mathbf{CM}(X)$ is the geometric realization of the nerve of the category $\mathbf{CM}(X)$, and $B\mathrm{PL}(X)$ is the classifying space of the simplicial group $\mathrm{PL}(X)$. The philosophical outcome is that the combinatorial homotopy of $\mathbf{CM}(X)$ can serve as an organizer of some known results and problems involving triangulations. This includes old Alexander stuff, connections with the Oda Conjecture in toric geometry, the problem of local formulas for characteristic classes, and combinatorial models of TQFT.

Seminariets hemsida: http://www.math.kth.se/~kozlov/seminar.html.

Tid och plats: Onsdagen den 8 november kl. 10.15 i seminarierum 3733, Institutionen för matematik, KTH, Lindstedtsvägen 25, plan 7.

MATEMATIKENS ÅR 2000

John Conway:
FRACTRAN — a logical programming language

John Conway holds the John von Neumann Distinguished Professor of Mathematics at Princeton University, USA. He is a former professor of mathematics at Cambridge University and remains an honorary fellow of Caius College. He is a fellow of the Royal Society and has received the Polya Prize of the London Mathematical Society. Recently he has been awarded the 1998 Frederic Esser Nemmers Prize in Mathematics from Northwestern University and the 2000 Steele Prize in Mathematical Exposition from the American Mathematical Society.

Abstract: Here is a sample of FRACTRAN: Start from the number 2, and then repeatedly multiply the integer you have at any stage by the first of the fractions:

$$\begin{array}{cccccccccccccc} \frac{17}{91} & \frac{78}{85} & \frac{19}{51} & \frac{23}{38} & \frac{29}{33} & \frac{77}{29} & \frac{95}{23} & \frac{77}{19} & \frac{1}{17} & \frac{11}{13} & \frac{13}{11} & \frac{15}{14} & \frac{15}{2} & \frac{55}{1} \\ =A & =B & =C & =D & =E & =F & =G & =H & =I & =J & =K & =L & =M & =N, \text{ say.} \end{array}$$

That gives an integral answer. Watch for the powers of 2 — after 2 itself, they are

$$2^2 \quad 2^3 \quad 2^5 \quad 2^7 \quad 2^{11} \quad 2^{13} \quad 2^{17} \quad 2^{19} \quad 2^{23} \quad 2^{29} \quad 2^{31} \quad \ldots\,.$$

I trust you get the pattern? I shall explain how this comes about, and show how you can program all possible computations in FRACTRAN.

Tid och plats: Måndagen den 13 november kl. 11.15 – 12.00 i sal E1, KTH, Lindstedtsvägen 3, b.v.

SEMINAR IN THEORETICAL PHYSICS

Rikard von Unge:
S-duality of noncommutative gauge theory and noncommutative open string theory

Abstract: We examine several aspects of S-duality of four-dimensional noncommutative gauge theory. By making an explicit duality transformation of noncommutative gauge theory, we run into a puzzle which will be the starting point of the talk. This puzzle was resolved by the introduction of Noncommutative Open String Theory (NCOS). We will explicitly find the degrees of freedom responsible for the NCOS theory, both in the noncommutative gauge theory and in string theory. The talk will be introductory and no previous knowledge of the above-mentioned phenomena will be necessary.

Tid och plats: Tisdagen den 7 november kl. 13.15 i rum 4731, Fysikum, SU, Vanadisvägen 9.

ALGEBRAIC GEOMETRY SEMINAR

Tom Graber:
Hurwitz numbers and moduli of curves

Abstract: I will discuss a formula of Ekedahl, Lando, Shapiro, and Vainshtein, which relates the number of branched covers of the Riemann sphere to integrals of tautological classes on the moduli space of pointed curves. I will sketch a short proof of this formula and describe how it can be applied to deduce facts about the Chow groups of the moduli space. This is all based on joint work with Ravi Vakil.

Tid och plats: Torsdagen den 9 november kl. 15.15 i sammanträdesrum 3548, Institutionen för matematik, KTH, Lindstedtsvägen 25, plan 5.

DOKTORANDSEMINARIUM

Johan Andersson: Riemanns zeta-funktion
och Poissons summationsformel för $SL(2, Z)$

Sammanfattning: Jag kommer att diskutera vad motsvarigheten till Poissons summationsformel blir på $SL(2, Z)$. Klassiskt sett har Selbergs spårformel ansetts vara den naturliga analogin. Jag kommer att presentera en ifrån min utgångspunkt naturligare summeformel. Formeln kan ses som en generalisering av såväl Kuznetsovs summeformel, som Selbergs spårformel. Jag kommer att visa hur formeln är en naturlig utgångspunkt för att bevisa resultat av Motohashi om fjärdemomentet av Riemanns zeta-funktion på den kritiska linjen. Jag kommer att diskutera hur situationen ser ut för $2n$-te momentet där motsvarande teori för $SL(n, Z)$ väntas dyka upp.

Tid och plats: Fredagen den 10 november kl. 15.15 i rum 306, hus 6, Matematiska institutionen, SU, Kräftriket, Roslagsvägen 101.

WAVELETSEMINARIUM

Dževad Belkić:

Noise reduction in generic signals, spectra and images

Abstract: In spectral analysis the main goal is to determine precisely all the quantification parameters of each peak from measurements of time signals that are most frequently imbedded in noise. The Fast Fourier Transform (FFT) of 'noisy' time signals leads to severely distorted frequency spectra. This is due to the linearity of FFT, which amplifies additive noise. By contrast, the Fast Padé Transform (FPT), defined as a unique quotient of two polynomials, is capable of significantly reducing random noise from any spectra by using its nonlinearity to effectively manipulate with noise. Such an achievement is possible because FPT is a generic parameter estimator which provides the position, width and magnitude of every spectral line from the first principles of the Cauchy calculus of residue without resorting to any fitting. To reduce noise, we take advantage of the computed table of positions, widths and magnitudes of each peak in a spectrum before we construct it in a final form. Random noise is a stochastic phenomenon which cannot be adequately described by any mathematical model with well-controlled outputs. Therefore, noise peaks in a spectrum will be exceedingly sensitive to any alteration in a given method. In FPT, we monitor the sensitivity of the peak parameters relative to the signal length. Alternatively, we supplement the original signal with an additional $\sim 10\,\%$ of the known noise whose realizations can be varied to perform the sensitivity test of the spectral features and to look for the coherence pattern recognition. Generally, we observe that the genuine, physical peaks are stable to within a prescribed threshold accuracy. Unstable peaks are identified as noise and removed from the spectral representation, which is the Heaviside partial fraction expansion. Both mathematical and physical justification for this strategy within FPT can be found. According to the Fröbenius-Froissart theorem for a rational function, as the one encountered in FPT, changing the signal length, while passing from the diagonal to para-diagonal elements in a two-dimensional Padé table, leaves the true poles virtually unaltered, whereas the spurious, extraneous resonances exhibit great instability. Physically, there is an analogy between noise and a background contribution in resonance scattering phenomena in physics, whose stabilization method is reminiscent of the above-mentioned peak sensitivity test. The power of the present noise reduction technique has been proven in many examples where FPT was effectively coupled to experiments in our pursuit to improve resolution. In these illustrations, we used measured 'noisy' time signals to compute magnitude or absorption spectra and always obtained greatly enhanced resolution with FPT relative to FFT, simultaneously achieving a considerable improvement of the signal-to-noise ratio. Supporting evidence will be presented at the seminar for a number of spectroscopic and imaging data from magnetic resonance physics.

Tid och plats: Torsdagen den 16 november kl. 10.30 – 11.15 i rum 1537, NADA, KTH, Lindstedtsvägen 3, plan 5.

MONEY, JOBS

Columnist: Pär Holm, Department of Mathematics, SU. E-mail: pho@matematik.su.se.

Info = information. This will be given and repeated until obsolete. Rely on other sources as well.
BBKTH = Bulletin Board at the Department of Mathematics, KTH.
BBSU = Bulletin Board at the Department of Mathematics, SU.

Unless stated otherwise, a given date is the last date (e.g. for applications), and the year is 2000. A number without an explanation is a telephone number.

Standard information channels

1. A channel to information from TFR: http://www.tfr.se.
2. A channel to information from NFR: http://www.nfr.se.
3. A channel to information from the European Mathematical Society: http://www.emis.de.
4. A channel to information from the American Mathematical Society: http://www.ams.org.
5. KTH site for information on funds, etc., weekly: http://www.kth.se/aktuellt/stipendier/.
6. Stockholm University site for information on funds: http://apple.datakom.su.se/stipendier/.
7. Umeå site for information on funds: http://www.umu.se/umu/aktuellt/stipendier_fond_anslag.html.
8. Job announcement site: http://www.maths.lth.se/nordic/Euro-Math-Job.html. This is run by the European Mathematical Society.
9. KTH site for information on research: http://www.admin.kth.se/CA/extrel/index/forsk.html.

New information

Jobs, to apply for

10. Institutionen för matematik vid KTH söker tre universitetslektorer i matematik, 16 november. Info: Ari Laptev, 08-790 62 44, laptev@math.kth.se. Web-info: http://web.kth.se/aktuellt/tjanster/Anst/Univlekt_Matematik.html.
11. Institutionen för matematik vid KTH söker en universitetslektor i optimeringslära och systemteori, 16 november. Info: Anders Lindquist, 08-790 73 11, alq@math.kth.se. Web-info: http://web.kth.se/aktuellt/tjanster/Anst/Univlekt_Matematik.html.
12. Chalmers Finite Element Center vid Chalmers tekniska högskola söker doktorander i tillämpad matematik och tekniska beräkningar, 22 november. Web-info: http://www.chalmers.se/HyperText/Lediga/8DranderMatte.html.
13. Institutionen för fysik och matematik vid Mitthögskolan i Sundsvall söker tre doktorander i systemanalys och matematisk modellering, 22 november. Info: Mårten Gulliksson, 070-623 78 30, marten.gulliksson@ind.mh.se. Web-info: http://www.mh.se/jobb/FSCN001026-3.html.
14. Matematiska institutionen vid SU söker två forskarassistenter i matematik, 8 december. Info: Torbjörn Tambour, 08-16 45 16, torbjorn@matematik.su.se, eller Bibi Pehrson, 08-16 22 92, bib.pehrson@natkan.su.se. Web-info: http://www.matematik.su.se/matematik/jobb/Foassmatte00.html.

Old information

Money, to apply for

15. Stiftelsen för internationalisering av högre utbildning och forskning (STINT) utlyser bidrag för kortare utlandsvistelser för lärare eller forskare vid svenskt universitet, högskola eller forskningsinstitut, dock ej doktorander. Ansökan kan inlämnas fortlöpande under året, dock senast 8 veckor före den dag då utlandsvistelsen avses påbörjas. Web-info: http://www.stint.se/KPutlys.html.
16. Anslag ställs, från Knut och Alice Wallenbergs Stiftelse, till rektors för KTH förfogande för att "i första hand användas till bidrag för sådana resor, som bäst befordrar ett personligt vetenskapligt utbyte till gagn för svensk forskning. Bidrag skall främst beviljas till yngre forskare." Ansökan om resebidrag skall ställas till rektors kansli. Bidrag kan sökas när som helst under året. Info: se punkt 5 ovan.

(Continued on the next page.)

17. Nordisk Forskerutdanningsakademi (NorFA) finansierar nordiskt samarbete inom forskning och forskarutbildning genom dels personliga stipendier (mobilitetsstipendier och för deltagande i nationella forskarutbildningskurser), dels anslag till institutioner (forskarutbildningskurser, nordiska nätverk, gästprofessurer och workshops). Info: http://www.norfa.no.
18. Svenska Institutet (SI) utlyser kontinuerligt stipendier och bidrag för studier och forskning utomlands: stipendier för Europastudier, internationella forskarstipendier, Östersjöstipendier, Visbyprogrammet, m.m. Aktuell information om SI:s samtliga stipendiemöjligheter och ansökningshandlingar finns på SI:s hemsida: http://www.si.se.
19. Stiftelsen för internationalisering av högre utbildning och forskning (STINT) utlyser medel för att främja samarbete med universitet och högskolor i Republiken Korea (Sydkorea), Taiwan, Hongkong, Indonesien och Egypten. Ansökningar skall inlämnas minst 6 – 8 veckor före verksamhetsstarten, och medlen kan sökas löpande under året. Info: STINT, Skeppargatan 8, 114 52 Stockholm, 08-662 76 90. Web-info: www.stint.se.
20. Wenner-Gren Stiftelserna utlyser gästföreläsaranslag, avsedda att möjliggöra för svenska forskare eller institutioner att inbjuda utländska gästföreläsare. Anslag sökes av den inbjudande forskaren eller institutionen. Ansökan kan inlämnas när som helst under året. Web-info: http://www.swgc.org/.
21. NUTEK stipends for stay in research institutions (not universities) in Japan. Short or long periods. For persons with or almost with doctoral degree. Info: Kurt Borgne, 08-681 92 65, kurt.borgne@nutek.se. You can apply at any time.

Jobs, to apply for

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23. Försvarets radioanstalt (FRA) söker person med utbildning i matematik, datalogi eller liknande, gärna forskarutbildning, för arbete som kryptolog, 6 november. Svenskt medborgarskap är ett krav, och innan man anställs kommer registerkontroll att göras. Info: Anders Eriksson eller Ola Sommelius, 08-471 46 00. Web-info: http://www.stepstone.se/sok/ramme2.html?fs=finn&done=yes&sok=kundeid&id=47069. Se Bråket nr 34 sidan 8.
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26. Matematiska institutionen vid Linköpings universitet söker minst en universitetslektor i tillämpad matematik, 22 november. Info: Svante Linusson, 013-28 14 45, svlin@mai.liu.se, eller Arne Enqvist, 013-28 14 14, arenq@mai.liu.se. Web-info: http://www.info.liu.se/jobb/mera/LiU1313-00-32.html.
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29. Naturvetenskapliga forskningsrådet (NFR) utlyser en forskartjänst inom stokastiska processer, 15 december. Info: Natalie Lunin, 08-454 42 32. Web-info: se punkt 2 ovan.

GRADUATE COURSE (continued)

Dževad Belkić:
Resolution Enhancement in Signal and Image Reconstructions / continued
(Master equations in MRI and CT)

Dževad Belkić is Guest Professor in "Mathematical Radiation Physics" at Karolinska Institutet, Stockholm.

The credit of the course is 5 p. It is given on one day per week (Wednesday) from January 24, 2001, to March 21, 2001.

The course is given in room 1537, NADA, KTH, Lindstedsvagen 3, floor 5.

Literature:

1. A textbook by Frank Natterer, *The Mathematics of Computerized Tomography,* John Wiley & Sons, New York (1989).
2. A textbook by Dževad Belkić, *The Principles and Methods of Quantum Scattering Theory,* Institute of Physics Publishing Ltd. (Bristol, England), to appear in March 2001 [ISNP 0750304960] (http://bookmark.iop.org/bookpge.htm?ID=617983879-6410-59741210-D&book=493h).

Description of the course / continued
(Master equations in MRI and CT)

This course describes the implementation of the multi-dimensional fast Padé transform (FPT), which has recently been introduced for Magnetic Resonance Imaging (MRI) and Computerized Tomography (CT) by Belkić (Nucl. Instr. Meth. B. **154:** 220 – 246, 1999, [*1*]; *The Principles and Methods of Quantum Scattering Theory*, Institute of Physics Publishing Ltd., (Bristol, England), in press 2001, [*2*]). The FPT uses the nonlinear epsilon-algorithm of Wynn to accelerate the sequence of fast Fourier transforms (FFT) generated with signals of gradually increasing length $N = 2^m$ $(m = 0, 1, 2, \ldots)$. Here, several illustrations are given for one-, two- and three-dimensional numerical quadratures whose accuracy is controlled solely by the value of the signal length. We obtain the unprecedented numerical precision to within twelve decimal places with $N = 1\mathrm{K}$, which corresponds only to 1024 sampling points. Convergence of FPT to this level of spectacular accuracy is extremely fast, since barely 64 and 256 equidistantly sampled points can secure four and eight decimal places, respectively. This is expected to introduce major improvements into image reconstructions and computerized tomography, since merely post-processing Padé-Wynn acceleration of Fourier sequences of varying length N is capable of extracting more information from experimentally recorded data than any advance in hardware would ever be able to accomplish.

Within the last two decades, a novel non-invasive retrieval or reconstruction technique called Magnetic Resonance Imaging (MRI) has nowadays attained the status of an essential part of diagnostic radiology. Similar to ultrasound and X-ray Computerized Tomography (CT), the ultimate goal of MRI is to generate two-dimensional images of preassigned sections of the examined human body. *Inter alia,* there are two distinct aspects of MRI that are advantageous relative to other competitive methods: (i) an arbitrary orientation/position of the imaged area, and (ii) a large contrast among soft tissues. In addition to imaging static anatomy, many clinical tasks use MRI for imaging blood vessels without contrast agents, cardiac imaging, dynamic imaging of the musculoskeletal system as well as for measuring tissue temperature and recording diffusion in tissue. The common denominator in all the MR encoding techniques is radio frequency (RF) excitation of the sample and its collection of the magnetic fields that originate from the nuclear magnetic spins. The Larmor precession

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of these magnetizations around the external constant magnetic field, combined with the additional magnetic field gradients and RF pulses, generates a current in the surrounding receiver coil. This is the pathway through which the sample responds to the perturbation after which deexcitation of the sample takes place leading to an echo with a composite electromagnetic time signal. Spectral and image analysis of this signal can reveal the types and the positions of the nuclei that are imaged within the sample. All the commercial software built into spectrometers and imaging scanners is based upon the fast Fourier transform (FFT).

The current key problem of MRI is long imaging times and insufficient spatial resolution. One of the reasons for such a situation is that advances in MRI have thus far been confined mainly to hardware upgrading. The other reason is that the commercial software is exclusively FFT, which is in the research field of signal and image processing known to be a low-resolution estimator. The situation can dramatically change to the better by complementing the FFT of the spectral and imaging devices with the FPT. All that FPT does is to accelerate convergence of a short sequence of merely a dozen of FFT's yielding an unprecedently improved resolving power of MRI.

The starting point of MRI is the definition of a reconstruction problem as an inversion of a 2D spatial Fourier integral whose result $S(k_x, k_y)$ is known as the experimentally measured data and the integrand $\rho(x, y)$ is the sought local spin density function or, equivalently, the spatial distribution of magnetization over the excited slice in the XOY plane:

$$c(t) \equiv S(k_x, k_y) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} dx dy \rho(x, y) \mathrm{e}^{2i\pi(k_x x + k_y y)}, \tag{1}$$

where the momenta $\{k_x, k_y\}$ and coordinate positions $\{x, y\}$ represent the two sets of the conjugate variables. The overall time (t) dependence of $S(k_x, k_y)$, symbolized by $c(t)$ in Eq. (1), stems from the fact that MRI conceives the momentum $\vec{k} = \{k_x, k_y\} \equiv \{k_x(t), k_y(t)\}$ as a time-varying function, which is determined by the linear gradient of the magnetic field $\vec{B}$ via

$$\vec{k}(t) = \gamma \int_0^t d\tau \vec{G}(\tau) \quad ; \quad \vec{G}(t) = \vec{\nabla} B(t), \tag{2}$$

where γ is the gyromagnetic factor. The ansatz (1) represents the so-called imaging equation, which is written here in a simplified form with neglected effects of inhomogeneity, susceptibility, chemical shift, diffusion, and relaxation time (T_2). The latter effect can be approximately included as an exponential factor, $\exp(-t/T_2)$, in the r.h.s. of Eq. (1) for those values of t that are not too much smaller than T_2. Here, as usual in the echo planar imaging (EPI), the component $G_y(t) = G_y(0) \equiv G_y$ is kept stationary, i.e., constant during the whole experimental recording, so that the integral for k_y is simplified as $k_y = \gamma G_y t$, thus yielding an overall multiplicative term, $\exp(-t/T_2) = \exp[-k_y/(\gamma G_y T_2)]$, which appears in front of the integral in Eq. (1).

Given a noiseless set of data, $S(k_x, k_y)$, its inverse Fourier transform, $F^{-1}[S(k_x, k_y)]$, can uniquely retrieve the effective spin density and, hence, arrive at the ultimate solution of the reconstruction problem, the searched 2D image:

$$\rho(x, y) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} dk_x dk_y S(k_x, k_y) \mathrm{e}^{-2i\pi(k_x x + k_y y)}. \tag{3}$$

It is well-known from e.g. the seminal work of Stig Ljunggren (J. Magn. Res. **54:** 338, 1983),

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that a simple and plausible visual interpretation of Eqs. (1) and (3) can now be given as follows. With increasing time t, the momentum vector $\vec{k}(t)$ moves in the 2D $\vec{k}$-space and thus traces a trajectory, $S(k_x(t), k_y(t))$, which is nothing but an envelope of the time signal, $c(t)$, also known as the free-induction decay (FID) rate in signal processing in Nuclear Magnetic Resonance (NMR) and Magnetic Resonance Spectroscopy (MRS). In the procedure of such a scanning of the $\vec{k}$-space, what one actually measures, in encoding through the EPI, is the value of the FID, $c(t) = S(k_x(t), k_y(t))$, at each point of the $\vec{k}$-trajectory, and this is the direct Fourier transform (1) of the spin density, $F[\rho(x, y)] = S(k_x, k_y)$. One of the obvious useful advantages of this analysis is that the corresponding inverse Fourier transform (3), $F^{-1}[S(k_x, k_y] = \rho(x, y)$, of the measured *noiseless* data, $S(k_x, k_y)$, reconstructs exactly the spin spatial density $\rho(x, y)$ and, hence, accomplishes the final task of MRI.

In practice, the examined sample is placed into an external static uniform magnetic field and exposed additionally to a linear gradient $\vec{G} = \{G_x, G_y\}$ through a sequence of radio-frequency pulses. These gradients produce the vector $\vec{k} = \{k_x, k_y\}$ in the momentum $\vec{k}$-space. A response of the examined sample to this combined external perturbation is assumed to be faithfully recorded in the surrounding receiver coils. Such a measured FID, $c(t)$, is precisely an equidistantly sampled 2D matrix or table $S(k_x(t), k_y(t))$ which is Fourier inverted to yield the sought image. This is the basis of the encoding via the EPI which is built in all commercial MRI scanners.

In digital Fourier-based processing within MRI one uses the discretized version of all the variables on equidistant grids:

$$k_{x,y} = n_{x,y}\Delta k_{x,y} \quad ; \quad x = m_x\Delta x \ , \ y = m_y\Delta y \quad ; \quad t = n_t\Delta t \tag{4}$$

$$\Delta k_{x,y} = 1/L_{x,y} \quad ; \quad \Delta x = L_x/(2N) \quad , \quad \Delta y = L_y/(2N) \quad ; \quad \Delta t = T/N \ ; \ N = 2^m. \tag{5}$$

Here we have $-N \le n_{x,y} \le N-1$, $-N \le m_{x,y} \le N-1$, $0 \le n_t \le N-1$, where T is the total acquisition time, Δt is the sampling time, and N is the signal length chosen in the Cooley-Tukey form of a nonnegative integer power of two, $N = 2^m$ $(m = 0, 1, 2, 3, \ldots)$. The lengths L_x and L_y are the so-called fields of view (FOV) in the x and y direction, respectively. Every set $\{c_n\}$ of the received digitized signal points, $c_n \equiv c(t_n)$, at different times $t = t_n \equiv n\Delta t$ from an imaged sample $S_{n_x,n_y} \equiv S(n_x\Delta k_x, n_y\Delta k_y)$, is modelled by a 2D coordinate representation of the Fourier transform (1) of the spatial magnetization distribution over the excited slice, $\rho_{n_x,n_y} \equiv \rho(n_x\Delta x, n_y\Delta y)$. Since T and $L_{x,y}$ are all finite, the above imaging equations must be modified to represent the definite two-sided symmetric double Fourier integrals, such as:

$$S(k_x, k_y) = \frac{1}{L_xL_y}\int_{-L_x}^{L_x}\int_{-L_y}^{L_y} dxdy\rho(x, y)\mathrm{e}^{2i\pi(k_xx+k_yy)}, \tag{6}$$

and likewise for Eq. (3). The Discrete Fourier Transform (DFT) is a variant of the simple trapezoidal quadrature rule for integral (6) with the Fourier grid points selected for both conjugate variables, the two-dimensional momentum $\vec{k} = (k_x, k_y)$ and position $\vec{r} = (x, y)$ from Eqs. (4) and (5):

$$S(k_x, k_y) = \frac{1}{2^{2m+4}}\sum_{m_x=-2^m}^{2^m-1}\sum_{m_y=-2^m}^{2^m-1}\rho_{m_x,m_y}\mathrm{e}^{2i\pi(m_xk_x\Delta x+m_yk_y\Delta y)}, \tag{7}$$

and

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$$\rho(x,y) = \sum_{n_x=-2^m}^{2^m-1} \sum_{n_y=-2^m}^{2^m-1} S_{n_x,n_y} \mathrm{e}^{-2i\pi(n_x x\Delta k_x + n_y y\Delta k_y)}. \tag{8}$$

The reason for resorting to the Fourier grid sampling in the $\vec{k}$-space is motivated by the possibility of using the Fast Fourier Transform (FFT). The latter is obtained from DFT by employing the special signal length $N = 2^m$ together with the Cooley-Tukey algorithm which, in e.g. the one-dimensional case, can significantly reduce the number of multiplications from N^2 to only $N\log_2 N$. The FFT is a fast algorithm for a fixed signal length, N, but the sequence of FFT's created with different values of N is slowly converging with increasing N. The convergence rate of e.g. one-dimensional FFT is only $1/N$, pointing to a basically low efficiency of the Fourier method regarding the augmentation of the signal length. This is presently illustrated with several typical examples of one-, two- and three-dimensional (1D, 2D & 3D) numerical quadratures (see below). Another difficulty with FFT is in dealing with signals embedded in noise as those routinely encountered in experimentally measured FID rates. Due to its linearity, FFT transfers the "noisy" part of the total signal directly to spectra or images causing a blur. Furthermore, when noise is present, the inequality $F^{-1}[S(k_x, k_y)] \neq \rho(x,y)$ holds true and the Fourier method is not the correct technique for image reconstruction. Of course, under ideal conditions the spins of protons can be quite accurately manipulated to be amenable to a reliable spectral analysis by FFT. But, whenever the ideal conditions are not secured, as is most frequently the case in practice, FFT is not an adequate solver of inverse problems in MRI.

These limitations of FFT can be circumvented by e.g. the Fast Padé Transform (FPT) [*1*]. For example, we have recently shown [*2*] that the FPT for 1D signal processing has the same resolving power as FFT, but uses significantly shorter data records. Equivalently, the FPT achieves higher resolution than FFT for the same value of the signal length N. Moreover, we have demonstrated in Ref. [*2*] that the slow convergence of FFT with increasing values of N can be significantly improved by FPT (see below). This is because FPT is, by definition, an accelerator of slowly converging series or sequences with an enhanced convergence rate relative the original expansions. This feature alone is useful in practice since it suggests that shorter acquisition times may suffice for FPT relative to FFT to achieve the required accuracy. The FPT is a nonlinear processor and, as such, is capable of considerably reducing the background noise. This has successfully been demonstrated in 1D signal processing of spectra encountered in physics as well as chemistry, and a similar conclusion is expected to hold true in image reconstructions, as well.

The master imaging equation (1) can be modified to take advantage of FPT. As it stands, Eq. (1) is a bivariate polynomial. However, two-variable polynomial approximations to surfaces are good only for relatively smooth regions, but are otherwise inadequate for discontinuous functions or areas with pronounced periodicity, sharp variations and integrable singularities. In such cases, rational functions as convenient nonlinear approximations prove to be more accurate. Among them the FPT exhibits several unique advantages as it yields the optimally accurate results with the least computational effort. The FPT is an accelerator of convergence of FFT with respect to the increasing size of the signal length, N. In practice, this amounts to considering a sequence of partial sums $\{\rho_\mu(x,y)\}_{\mu=0}^{m}$ whose members are all computed via FFT according to:

$$\rho_\mu(x,y) = \sum_{n_x=-2^\mu}^{2^\mu-1} \sum_{n_y=-2^\mu}^{2^\mu-1} S_{n_x,n_y} u_x^{n_x} u_y^{n_y} \quad ; \quad (\mu = 0,1,2,3,\ldots,m), \tag{9}$$

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with the property $\lim_{\mu\to m}\rho_\mu(x,y)=\rho(x,y)$. Here, $u_x=\exp(2i\pi x\Delta k_x)$ and $u_y=\exp(2i\pi y\Delta k_y)$ where $\Delta x,\Delta y$ and $\Delta k_{x,y}$ are taken from Eqs. (4) and (5), but with an important replacement of the full signal length $N=2^m$ by the partial one, $N=2^\mu$. Several linear or nonlinear techniques can be utilized to improve the convergence rate of the sequence $\{\rho_\mu(x,y)\}_{\mu=0}^m$. We presently choose one of the nonlinear methods, the FPT, in its operational form as given in Ref. [*1*]. The FPT, denoted by $R(x,y)$, as an approximation to the true spin density, $R(x,y)\approx\rho(x,y)$, is a ratio of two polynomials, $R(x,y)=P(u_x,u_y)/Q(u_x,u_y)$. Both polynomials, P and Q, can be obtained explicitly by using the r.h.s. of Eq. (8) to set up and solve a system of linear equations. This is important in quantification of spectra, as in 1D and 2D versions of NMR and MRS. However, for MRI one does not necessarily need such quantifications (peak finding) of structures and features of images, so that in here the explicit knowledge of the polynomials P and Q in FPT is not required. Therefore the well-known Wynn epsilon (ϵ) recurrence relation among four neighbouring elements of the Padé 2D table can be used to generate the ratio $P(u_x,u_y)/Q(u_x,u_y)$ as a whole without any separate computations of the numerator (P) and denominator (Q) polynomials. This recursive ϵ-algorithm of Wynn is stable as well as robust and, moreover, remarkably simple for straightforward programming:

$$\epsilon_{\nu+1}^{(\mu)}=\epsilon_{\nu-1}^{(\mu+1)}+\frac{1}{\epsilon_\nu^{(\mu+1)}-\epsilon_\nu^{(\mu)}} \quad ; \quad (\nu,\mu>0), \tag{10}$$

where the sole initialization is provided by the sequence of the partial sums, $\{\rho_\mu(x,y)\}_{\mu=0}^m$, from Eq. (9):

$$\epsilon_{-1}^{(\mu)}=0 \quad ; \quad \epsilon_0^{(\mu)}=\rho_\mu(x,y) \quad , \quad (\mu=0,1,2,\ldots,m). \tag{11}$$

The recursion (10) and the initialization (11) are carried out at the fixed point $(x,y)=(n_x\Delta x,n_y\Delta y)$. The computation is repeated for any other Fourier mesh points to scan the entire area within the spatial boundaries L_x and L_y. Thus, at a selected position (x,y), one first generates the ϵ-sequence, $\{\epsilon_\nu^{(\mu)}\}$, and then monitors its convergence with respect to the even-numbered subscripts only, $\nu=2j$ $(j=1,2,3,\ldots)$. The limit of this latter subsequence of the ϵ-arrays represents the estimate of the FPT for $\rho(x,y)$. The FPT is a low-storage method, since it involves only 1D arrays. The ϵ-entries are defined as two-dimensional matrices *per se,* but nevertheless the Wynn recursion (10) remains only a very simple 1D algorithm. This is because some intermediate results can safely be overwritten without affecting the possibility to obtain the auxiliary sequence $\{\epsilon_{2j-1}^{(\mu)}\}$ together with the main result $\{\epsilon_{2j}^{(\mu)}\}$ at each point (x,y). To take advantage of FFT, the sequence of partial sums, $\{\rho_\mu(x,y)\}_{\mu=0}^m$, from Eq. (9) is computed only at the Fourier grid points for (x,y). Of course, the Padé approximant is not necessarily restricted to the Fourier mesh for (x,y) and, in principle, any other spatial sampling can be selected. However, in such a case the computation of the partial sums, $\{\rho_\mu(x,y)\}_{\mu=0}^m$, would have the scaling of DFT with the increased N rather than that of FFT. In practice, this is deemed unnecessary. Note that the above presentation has been explicitly given in the 2D case, but the FPT extends directly to e.g. three-dimensional imaging. To this end, all one needs is to replace the data sets $\{S_{n_x,n_y};\rho_\mu(x,y);(k_x,k_y)\}$ by $\{S_{n_x,n_y,n_z};\rho_\mu(x,y,z);(k_x,k_y,k_z)\}$, respectively. Here, S_{n_x,n_y,n_z} is the direct extension of Eq. (7) to the 3D case with an additional sum over m_z having -2^m and 2^m-1 for the lower and upper limits, respectively, and using $\rho_{n_x,n_y,n_z}\exp[2i\pi(m_xk_x\Delta x+m_yk_y\Delta k_y+m_zk_z\Delta k_z)]$ in lieu of $\rho_{n_x,n_y}\exp[2i\pi(m_xk_x\Delta x+m_yk_y\Delta k_y)]$. The lattice spin density ρ_{n_x,n_y,n_z} follows by a straightforward extension of Eq. (9) to 3D though introduction of one more sum over n_z running

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from -2^μ to $2^\mu - 1$ and replacing $S_{n_x,n_y} u_x^{n_x} u_y^{n_y}$ by $S_{n_x,n_y,n_z} u_x^{n_x} u_y^{n_y} u_z^{n_z}$ where $u_z = \exp(2i\pi z \Delta k_z)$. Then the 3D version of FPT follows by using again the ϵ-algorithm of Wynn in exactly the same form as in Eq. (10), which is invariant to the considered number of dimensions. In fact, the only thing which changes in going from 2D to 3D variants of FPT is the initialization (11) in which the 2D partial sums $\rho_\mu(x,y)$ should be replaced by their 3D counterparts, $\rho_\mu(x,y,z)$. Moreover, the formulation of FPT encompassing an arbitrary dimension, from the simplest 1D case to nD with any positive finite integer n, for the purpose of versatile and generic applications in e.g. evaluations of multiple series and integrals, proceeds along a similar pathway as outlined previously in Ref. [*1*]. By the same token, the imaging equations (6) and (7) are automatically carried over to any finite number of dimensions.

In the recent textbook [*2*], we extended FPT to encompass the Computerized Tomography (CT) by using the Padé approximant P/Q as an optimal filter and also as an accelerator of two-dimensional quadratures. In so doing, we implemented an *explicit* version of the attenuated Radon Transform (RT), $R_\mu(\omega, p)$, where the Padé polynomial quotient P/Q is advantageously employed to enhance convergence of the inherent Riemann sum, which is a simple trapezoidal-type numerical integration. In CT, given the map of the attenuation coefficients $\mu(x)$ of tissue, one measures the radiation flux or emission data, $g(\omega, p)$, defined by the attenuated RT:

$$g(\omega, p) \equiv (R_\mu f)(\omega, p) = \int_{x \cdot \omega = p} f(x) \rho^-(\omega^\perp, x) \mathrm{d}x, \tag{12}$$

where $\mathrm{d}x$ is the two-dimensional Lebesgue measure restricted to the line $x \cdot \omega = p$, the symbol $\omega \cdot x$ denotes the inner product, ω is a directional two-component angle, $\omega = (\cos\phi, \sin\phi)$, $\omega^\perp = (-\sin\phi, \cos\phi)$, and $\rho^\pm(\omega, x) = \exp(\pm \int_0^\infty dt \mu(x + t\omega)$, with t being a scalar. The final mathematical goal in CT is to numerically invert Eq. (12) for $g(\omega, p)$ and obtain the integrand $f(x)$, which is the activity distribution. In computations, we employ the following explicit inversion formula for $f(x)$:

$$f(x) = -\frac{1}{4\pi} \operatorname{div} Re \int_{S^1} d\omega \omega \rho^+(\omega^\perp, x) g_\mu(\omega, x \cdot \omega), \tag{13}$$

where S^1 is a circle. Here, $g_\mu(\omega, p) = (\mathrm{e}^{-h} H \mathrm{e}^h g)(\omega, p)$, with $2h \equiv 2h(\omega, p) = (I + iH) R_\mu(\omega, p)$, $i = +\sqrt{-1}$, where I and H are the identity and the Hilbert transform, $(Hg)(\omega, p) = (1/\pi)\mathcal{P}\int_{R^1} d\tau g(\omega, \tau)/(p - \tau)$, respectively, and the symbol $\mathcal{P}$ stands for the usual Cauchy principal value. Our implementation of Eq. (13) for $f(x)$, when computationally refined through FPT, yields much more accurate results than the conventional back-projection technique for numerical inversion of Eq. (12) for given functions $g(\omega, p)$ and $\rho^-(\omega^\perp, x)$.

Critical to MRI is the accuracy and speed of computations as well as stability and robustness in the computations. Accuracy is the weakest point of FFT, but the other three mentioned features are not a problem. It is these three latter features which FPT shares with FFT. One of the novel features brought to the field of MRI by FPT is its improved accuracy which is enhanced by *orders of magnitude* relative to the conventional FFT as highlighted below. To this end we have carried out a large number of tests, and a detailed analysis has been reported in Ref. [*2*]. Since Eq. (3) is a direct quadrature for $\rho(x,y)$ with the known integrand $S(k_x, k_y)$, the performance of FPT will be most clearly tested on some exactly solvable double integrals. To proceed towards this goal, we consider evaluation of the following triple finite Fourier-type integral:

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$$\int_{a_1}^{b_1}\int_{a_2}^{b_2}\int_{a_3}^{b_3} dxdydz \mathrm{e}^{i\vec{q}\cdot\vec{r}}F(x,y,z)$$
$$= \Delta_1\Delta_2\Delta_3 \sum_{m_1=0}^{2^m-1}\sum_{m_2=0}^{2^m-1}\sum_{m_3=0}^{2^m-1} W_{m_1,m_2,m_3}^{m_x,m_y,m_z} F(a_1+m_1\Delta_1, a_2+m_2\Delta_2, a_3+m_3\Delta_3), \qquad (14)$$

where $\Delta_j = (b_j - a_j)/2^m$, $\vec{q} = (q_x, q_y, q_z)$, $\vec{r} = (x, y, z)$, $W_{m_1,m_2,m_3}^{m_x,m_y,m_z} = \mathrm{e}^{i\pi(m_x m_1 + m_y m_2 + m_z m_1)/2^{m-1}}$, and $m_j = 0, 1, 2, \ldots, 2^m$ $(j = 1, 2$ and $3)$. The special 1D and 2D cases of Eq. (14) are obtained by suppressing $\{y, z; \Delta_{2,3}\}$ and $\{z; \Delta_3\}$, so that the resulting integrals over the functions $F(x)$ and $F(x, y)$ finally lead to the single and double summation, respectively. As it stands, the threefold quadrature in Eq. (14) is replaced by the Riemann sum which, in the case of convergence, gives the exact result as m reaches its infinitely large value. Both 1D and 2D cases of Eq. (14) are of interest to Ion Cyclotron Resonance (ICR) mass spectroscopy, NMR and MRS. The 2D and 3D cases are important for MRI, whose main equation (3) for two dimensions belongs to the category of Eq. (14). In practice, we use FFT in Eq. (14) for $m = 0, 1, \ldots, 10$. Here, for compactness of presentation, we shall give the results that correspond to the origin of the discretized momentum $q_{x,y,z}$, that is $(n_x, n_y, n_z) = (0, 0, 0)$. Our finding is depicted in the displayed Table for a set of the selected 1D-, 2D- and 3D-quadratures. The columns headed by the labels 'Fourier' and 'Padé' represent the results of FFT and FPT, respectively. The data of FPT are obtained from the FFT sequence of different length, $N = 2^m$ $(m = 0, 1, 2, \ldots, 10)$, i.e., $N = 1, 2, 4, 8, 16, 32, 64, 128, 256, 512, 1024$. This latter FFT sequence is accelerated by the Wynn's ϵ-algorithm (10) and (11) throughout the 3D grid (n_x, n_y, n_z), and the results are displayed in the shown Table at the selected point $(n_x, n_y, n_z) = (0, 0, 0)$ to avoid dealing with complex numbers. Of practical importance is to emphasize that the speed of FPT is proportional to that of FFT, since the Wynn recursion (10) takes no time at all. This is the direct consequence of the present way of forming the partial sums S_m via the prescription $S_m \equiv \sum_{k=0}^{2^m-1} c_k$ rather than through $S_m \equiv \sum_{k=0}^{m-1} c_k$. Clearly, such an approach reduces tremendously the number of terms in the set of the partial sums, $\{S_m\}$. The results of FFT exhibit poor accuracy and unfavourable convergence properties with the increasing number N of the integration points, despite the simple functions selected in all the 1D, 2D and 3D quadratures displayed in the Table. By contrast, the FPT is seen in the same Table to be highly satisfactory, since its convergence is indeed impressively fast and the achieved improvement in accuracy relative to FFT is spectacular. For example, using $N = 1024$, the Fourier method barely secures one or at most two decimal places relative to 12 exact decimal places obtained by the Padé approximant. Even $N = 256$ suffices for the Padé method to yield at least seven decimal places of accuracy. The Table deals with $N^m \leq 1024$ $(m \leq 10)$, but we checked that the results of FPT for $N = 1024$ agree with those for $N > 1024$ at least to within twelve exact decimal places. (See note on page 18.) Yet, this essential improvement is obtained with no explicit computations of the quadrature themselves, but rather through post-analysis of the results of the trapezoidal numerical integrations. These latter results, therefore, inherently contain all these exact twelve decimal places that are masked by the straightforward addition of partial sums. It is fascinating that such a negative effect of simple additions is efficiently cancelled by the powerful procedure of Padé. This means that in e.g. ICR, NMR, MRS, MRI or CT, all one needs is a sequence of FFT's or attenuated RT's, to arrive at the unprecedented high-resolution with a simple post-processing through the Padé-Wynn recursion. *This is the essence of our FPT.* Such a procedure yields not only

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high resolving power, but also a considerably improved signal-to-noise ratio (SNR) as the Padé-Wynn acceleration effectively uses shorter FFT sequences. Implemented within MRI, the FPT yields images that are brighter and sharper relative to the conventional FFT software. For example an MRI scan with $N = 1\text{K} = 1024$ requires eight minutes of real time resulting in currently the most sharply resolved images via FFT, but the brightness will be inferior to the scan with $N = 512$. This is because doubling the number of sampling points would necessarilly invoke more noise and, hence, worsen the SNR. By comparisons, within approximately the same acquisition time T, the images from FPT post-processed with a Fourier sequence for $N \leq 512$ preserve the best features of FFT, i.e., its sharpness for $N = 1024$ as well as its brightness at $N = 512$.

To summarize, in Ref. [*1*] (op. cit.), we have introduced the Fast Padé Transform (FPT) for multidimensional signal and image processing. Unlike some recent versions of the Padé approximant employing the band-limited decimated time signals, the FPT does not require any windowing and, hence, the entire Nyquist intervals can be scanned in the most general, multi-dimensional case. Operationally, the FPT accelerates a given sequence of Fast Fourier Transforms (FFT) that are created with the same signal of varying length, N. The original sequence of FFT's converges slowly with increasing N and this can be spectacularly accelerated in FPT by means of the Wynn recursive algorithm. The evidence for the unprecedented accuracy within twelve decimal places achieved using only $N = 512$ or $N = 1024$ points is provided in the present illustration, which gives several examples in the case of one-, two- and three-dimensional quadratures. This is only a fraction of a more general experience we gained thus far with FPT which is, therefore, expected to be the method of choice particularly in one- and two-dimensional magnetic resonance spectroscopy (MRS) as well as in two- and three-dimensional magnetic resonance imaging (MRI). Such an expectation is based upon the established unique collections of the features of FPT, namely its accuracy, efficiency, robustness and simplicity that will be thoroughly analysed during this course.

Note: In Bråket No. 34, p. 13, the factor 3/4 multiplying $\sin^2 u$ in the 1D case should read 9/16. The 1D results reported there were obtained by truncating the trapezoidal sum at the upper limit 2^m, whereas the corresponding present findings are for the value $2^m - 1$ of that limit. No difference within twelve decimal places is found between these two runs of FPT for the converged values of $m \geq 8$, i.e. $N = 2^m \geq 256$. The 2D and 3D cases have been considered with the upper summation index $2^m - 1$ in the trapezoidal quadratures in both Nos. 34 and 35 of Bråket.

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ONE-, TWO- AND THREE-DIMENSIONAL FOURIER & PADÉ TRANSFORMS:

The significant figures are underlined, $\underline{1.23}4567\ldots$**(02)**, and **(02)** shows the number of the exact decimals. The results are for the origin of momentum $(q_x, q_y, q_z) = (n_x \Delta q_x, n_y \Delta q_y, n_z \Delta q_z)$ with $(n_x, n_y, n_z) = (0, 0, 0)$.

ONE DIMENSION (1D); $\int\limits_{R^1} dx \mathrm{e}^{2i\pi q_x x} F(x)$:

$F(x)$	$x(1-\frac{9}{16}\sin^2 x)^{-1/2}$	$x(1-\frac{9}{16}\sin^2 x)^{-1/2}$	$\frac{\cos(11x/4)}{1+x^2/4}\,\mathrm{e}^{5/4-x}$	$\frac{\cos(11x/4)}{1+x^2/4}\,\mathrm{e}^{5/4-x}$
$N \setminus R^1$	$x \in [0, \pi]$	$x \in [0, \pi]$	$x \in [0, 1]$	$x \in [0, 1]$
Exact	$\underline{6.003551456295}$	$\underline{6.003551456295}$	$\underline{0.740942995086}$	$\underline{0.740942995086}$
	Fourier	**Padé**	Fourier	**Padé**
16	$\underline{5.}695126318776$ (**00**)	$\underline{6.0}23177766031$ (**01**)	$\underline{0.}880907101501$ (**00**)	$\underline{0.74}3781874211$ (**02**)
64	$\underline{5.}926445171912$ (**00**)	$\underline{6.0035}55951607$ (**04**)	$\underline{0.}775705101784$ (**00**)	$\underline{0.7409}52222970$ (**04**)
256	$\underline{5.}984274885199$ (**00**)	$\underline{6.003551456295}$ (**12**)	$\underline{0.7}49619247067$ (**01**)	$\underline{0.74094299}5805$ (**08**)
1024	$\underline{5.}998732313521$ (**00**)	$\underline{6.003551456295}$ (**12**)	$\underline{0.74}3111166041$ (**02**)	$\underline{0.740942995086}$ (**12**)

TWO DIMENSIONS (2D); $\iint\limits_{R^2} dxdy \mathrm{e}^{2i\pi(q_x x + q_y y)} F(x, y)$:

$F(x,y)$	$(1+x^2+y^2)^{-2}$	$(1+x^2+y^2)^{-2}$	$\mathrm{e}^{3\lvert x+y-1\rvert/4}\cos\frac{x+y}{4/3}$	$\mathrm{e}^{3\lvert x+y-1\rvert/4}\cos\frac{x+y}{4/3}$
$N \setminus R^2$	$x\&y \in [0, \infty]$	$x\&y \in [0, \infty]$	$x\&y \in [0, 1]$	$x\&y \in [0, 1]$
Exact	$\underline{0.785398163398}$	$\underline{0.785398163398}$	$\underline{0.901478755468}$	$\underline{0.901478755468}$
	Fourier	**Padé**	Fourier	**Padé**
16	$\underline{0.}864132279031$ (**00**)	$\underline{0.78}6753599558$ (**02**)	$\underline{0.}953254902981$ (**00**)	$\underline{0.90}2177481063$ (**02**)
64	$\underline{0.8}04776456771$ (**01**)	$\underline{0.785398}758165$ (**06**)	$\underline{0.9}14234941779$ (**01**)	$\underline{0.90147}6883858$ (**05**)
256	$\underline{0.79}0223663747$ (**02**)	$\underline{0.78539816}0060$ (**08**)	$\underline{0.90}4656514100$ (**02**)	$\underline{0.90147875}9061$ (**08**)
1024	$\underline{0.78}6603346439$ (**02**)	$\underline{0.785398163398}$ (**12**)	$\underline{0.90}2272496733$ (**02**)	$\underline{0.901478755468}$ (**12**)

THREE DIMENSIONS (3D); $\iiint\limits_{R^3} dxdydz \mathrm{e}^{2i\pi(q_x x + q_y y + q_z z)} F(x, y, z)$:

$F(x,y,z)$	$\cos\frac{x+y+z}{4/3}$	$\cos\frac{x+y+z}{4/3}$	$\mathrm{e}^{-x-y-z}\,\frac{\sin(x+y+z)}{x+y+z}$	$\mathrm{e}^{-x-y-z}\,\frac{\sin(x+y+z)}{x+y+z}$
$N \setminus R^3$	$x, y\&z \in [0, 1]$	$x, y\&z \in [0, 1]$	$x, y\&z \in [0, \pi/2]$	$x, y\&z \in [0, \pi/2]$
Exact	$\underline{0.401767070469}$	$\underline{0.401767070469}$	$\underline{0.268816517890}$	$\underline{0.268816517890}$
	Fourier	**Padé**	Fourier	**Padé**
16	$\underline{0.}459965527350$ (**00**)	$\underline{0.40}1202019496$ (**02**)	$\underline{0.3}41001545981$ (**01**)	$\underline{0.27}4447992301$ (**02**)
64	$\underline{0.}416489780171$ (**00**)	$\underline{0.40177}3155873$ (**05**)	$\underline{0.2}85737649156$ (**01**)	$\underline{0.2688}49809913$ (**04**)
256	$\underline{0.4}05458211846$ (**01**)	$\underline{0.40176707}1568$ (**08**)	$\underline{0.27}2978880143$ (**02**)	$\underline{0.268816}557031$ (**06**)
1024	$\underline{0.40}2690504245$ (**02**)	$\underline{0.401767070469}$ (**12**)	$\underline{0.26}9852900426$ (**02**)	$\underline{0.268816517890}$ (**12**)