Steady states of anisotropic generalized Newtonian fluids

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Abstract

We consider the stationary flow of a generalized Newtonian fluid which is modelled by an anisotropic dissipative potential f. More precisely, we are looking for a solution $u : \Omega \to \mathbf{R}^n$, $\Omega \subset \mathbf{R}^n$, $n = 2, 3$, of the following system of nonlinear partial differential equations

$$\left\{ \begin{array}{rll} -\mathrm{div}\{T(\varepsilon(u))\} + u^k \frac{\partial u}{\partial x_k} + \nabla \pi & = g & \text{in } \Omega, \\ \mathrm{div}\ u = 0 \quad \text{in } \Omega, \qquad u & = 0 & \text{on } \partial\Omega. \end{array} \right. \tag{$*$}$$

Here $\pi : \Omega \to \mathbf{R}$ denotes the pressure, g is a system of volume forces, and the tensor T is the gradient of the potential f. Our main hypothesis imposed on f is the existence of exponents $1 < p \leq q_0 < \infty$ such that

$$\lambda(1 + |\varepsilon|^2)^{\frac{p-2}{2}} |\sigma|^2 \leq D^2 f(\varepsilon)(\sigma, \sigma) \leq \Lambda(1 + |\varepsilon|^2)^{\frac{q_0-2}{2}} |\sigma|^2$$

holds with constants $\lambda, \Lambda > 0$. Under natural assumptions on p and q_0 we prove the existence of a weak solution u to the problem (*), moreover we prove interior $C^{1,\alpha}$-regularity of u in the two-dimensional case. If $n = 3$, then interior partial regularity is established.