The multiplicity of positive solutions for boundary value problems to equations involving p-Laplacian

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We consider the boundary value problem

$$-\operatorname{div}\,\mathfrak{a}(\nabla u) = f(u) \quad \text{in } \Omega, \qquad u > 0 \quad \text{in } \Omega, \qquad u = 0 \quad \text{on } \partial\Omega,$$

in spherical shell $\Omega = B_{R_2} \setminus \overline{B}_{R_1} \subset \mathbb{R}^n$. Here $\mathfrak{a}(\sigma) \equiv \nabla \mathcal{A}(\sigma)$ and, roughly speaking, function $\mathcal{A}(\sigma)$ is "not more convex" then $|\sigma|^p$, and function $\mathcal{F}(s) = \int_0^s f(u)du$ is "more convex" then s^p. Thus, the model equation is

$$-\Delta_p u \equiv -\operatorname{div}\left(|\nabla u|^{p-2}\nabla u\right) = |u|^{q-2}u, \qquad q > p.$$

The phenomenon of multiplicity (in the case $p = 2$) was discovered by C.V.Coffman (1983) for $n = 2$, by Y.Y.Li (1990) for $n \geq 4$ and by J.Byeon (1997) for $n = 3$. More precisely, they proved that for $q > 2$ the number of rotationally nonequivalent solutions tends to infinity as $\varepsilon \equiv R_2/R_1 - 1 \to 0$.

We generalize these results to the case of arbitrary $p \in]1, +\infty[$ for $n \neq 3$. Also the "complementary" effect of multiplicity for fixed ε and large q is discovered. Some of results are new even for $p = 2$.

These results are generalized also for nonlinear Neumann boundary value problem

$$\begin{cases} -\Delta_p u + |u|^{p-2}u &= 0 \qquad \text{in } B_R \\ |\nabla u|^{p-2}\langle \nabla u; \mathbf{n}\rangle &= |u|^{q-2}u \quad \text{on } S_R \end{cases}, \qquad u > 0 \quad \text{in } B_R.$$

References

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