

ABSTRACTS

Kari Astala: **The nonlinear Fourier transform and inverse problems for degenerate elliptic equations**

Abstract: In inverse problems such as electric impedance tomography one aims to determine the structure of a body by electric measurements on the boundary. In mathematical terms, we are asked to determine the coefficients of a differential operator from the Dirichlet-to-Neumann data. In two dimensions, in particular, the most efficient methods are provided by the nonlinear Fourier transform, the study of solutions with exponential asymptotics. In this talk, based on joint works with Lassas and Päiväranta, we study the nonlinear Fourier transform for equations of divergence type, and use it to solve the inverse problem for operators with bounded or even degenerate coefficients, also identifying the limits of impedance tomography. In this generality the quasiconformal methods are indispensable.

Viviane Baladi: **Towards linear response for smooth one-dimensional dynamics – the Benedicks-Carleson case.**

Abstract: When a smooth one-parameter family f_t of dynamical systems admits for all (or many) t a unique SRB measure μ_t , it is natural to ask if the map $t \rightarrow \mu_t$ is also smooth. David Ruelle solved the case when the f_t are smooth uniformly hyperbolic, and, more recently, he proposed a formal series as a candidate for the linear response formula (the formula for the derivative of μ_t with respect to t). Suitable resummations of this series have been proved to give the linear response in the one-dimensional piecewise expanding (Baladi-Smania) and analytic nonrecurrent (Ruelle) cases (under a horizontality condition).

In the proofs, the solution α to the twisted cohomological equation (TCE)

$$v(x) = \alpha(f(x)) - f'(x) \alpha(x) \quad (v \text{ a smooth "horizontal" function})$$

plays a key part. In the essentially hyperbolic cases solved up to now, it is easy to show that the TCE has a bounded solution. The Collet-Eckmann situation is much more difficult. Recently, we were able to show, under an additional Benedicks-Carleson assumption, that the TCE admits a unique bounded solution, which is continuous. We shall explain the ideas in the proofs, and how we expect to exploit this result to obtain linear response. (Joint with D. Smania.)

Jean-Pierre Eckmann: **A topological glass**

Abstract: In this talk I will discuss work on Metropolis dynamics of triangulations on the sphere, with a weight which "punishes" a certain type of defect in these triangulations. This model is inspired, and makes some statements about the glassy dynamics of discs scattering on a torus.

Håkan Eliasson: **KAM for the NLS**

Abstract: We will present a work (joint with S. Kuksin) on the perturbation theory of finite-dimensional KAM-tori for the non-linear Schrödinger equation in dimension d (with periodic boundary conditions). The difficulties to apply a KAM-approach to this equation are substantial for $d \geq 2$, and its "Töplitz-Lipschitz"-property is essential to handle these difficulties. Our result improves on a previous result of Bourgain which gives the existence of quasi-periodic solutions for this equation.

Jacek Graczyk: **Schwarzian derivative in conformal dynamics**

Abstract: Schwarzian derivative is an important tool both in complex analysis and conformal dynamics. If an analytic map preserving the real line has a negative Schwarzian derivative then small circles intersecting the real line are contracted by inverse branches of the map. We prove that every smooth self map of a compact interval (circle) with all critical points non-flat and all periodic points repelling has a negative Schwarzian derivative after an analytic change of coordinates. The main idea is to solve a cohomological inequality in the class of essentially bounded measurable functions and then "smooth out" the solution. In the case of circle maps we need one more condition to solve the cohomological inequality due to integral obstructions.

Peter Jones: **Random Weldings: Part 2**

Abstract: This talk will be a continuation of the lecture given by Antti Kupiainen.

Mattias Jonsson: **Polynomial dynamics in two complex dimensions**

Abstract: Iterations of polynomial mappings constitute arguably the most basic holomorphic dynamical systems and yet can exhibit very rich behavior. In one dimension, using e.g. external rays, one can often trace back the recurrent dynamics to the dynamics at infinity, and the latter is completely described in terms of the Boettcher coordinate. As I will explain, in dimension two (and higher) the dynamics at infinity is much richer but can still impact the recurrent dynamics through the construction of invariant measures.

Antti Kupiainen: **Random weldings**

Abstract: Conformal welding provides a map between a class of closed curves on the plane modulo Möbius maps and a class of homeomorphisms of the unit circle. Using this we construct random curves from random homeomorphisms whose derivative is proportional to the exponential of the restriction of the two dimensional free field to the circle. This is joint work with K. Astala, P. Jones and E. Saksman.

François Ledrappier: **Linear drift and noncommutative ergodic theorems**

Abstract: For a regular cover of a compact Riemannian manifold, the linear drift of the Brownian motion is a natural quantity. It is related to the amenability of the covering group, to the volume growth and to the Liouville property. We shall review some of these properties and some questions.

Nikolai Makarov: **Ward identities in statistical mechanics and conformal field theory**

Abstract: Ward's identities and the related concept of the stress-energy tensor are standard tools in theoretical physics, in particular in conformal field theory. I will give a brief mathematical overview of these concepts, and outline two applications – to random normal matrices and to Schramm's SLE processes. Joint work with Y. Ameur and H. Hedenmalm, and with N.-G. Kang.

Feliks Przytycki: **Equilibria and analyticity of geometric pressure for iteration of rational maps**

Abstract: I shall consider pressure $P(t)$ of the potential function $-t \log |f'|$ for f a rational mapping on the Riemann sphere, and the (non)existence of equilibria and their statistical properties. I assume that f admits a nice (Markov-like) neighbourhoods of critical points, allowing nice inducing. This class of maps f includes topological Collet-Eckmann rational maps, non-renormalizable polynomials without indifferent periodic points, infinitely-renormalizable quadratic polynomials with a priori bounds. I will sketch the proof of the analyticity of $P(t)$ (between phase transition points) and deduce the analyticity of dimension spectrum of the Lyapunov exponent and the analyticity of $\beta(t)$ for f polynomial for the basin of infinity. These are results obtained jointly with Juan Rivera-Letelier.

Nessim Sibony: **On the dynamics of holomorphic foliations by Riemann Surfaces**

Abstract: Global properties of holomorphic foliations in CP^2 are poorly understood. It is still unknown whether every leaf clusters at a singular point. The subject is related to counting cycles for polynomial vector fields in the real plane. In this talk I will discuss unique ergodicity for such foliations. The main result is that for a foliation of degree d , with hyperbolic singularities and without algebraic leaves there is a unique positive harmonic current of mass one directed by the foliation. The class of foliations which satisfy the hypothesis is generic. This implies that appropriate averages on leaves have a unique limit, independent of the leaf and of the averaging process. If time permits we will discuss other aspects of the theory. This is joint work with J.E Fornæss.

Stas Smirnov: **Quasiconformal maps and harmonic measure**

Abstract: Many questions in complex analysis can be reduced to multifractal properties of the harmonic measure. Those are still poorly understood, and we will discuss possible approaches to them using quasiconformal maps and holomorphic motions. Partially based on joint work with Kari Astala and Istvan Prause.

Michael Sodin: **Weighted exponential approximation on the real axis**

Abstract: Suppose μ is a non-negative measure on \mathbf{R} so that $(t - i)^{-1}$ lies in $L^2(\mu)$. A classical problem in harmonic analysis is to find the minimal width of frequencies needed to approximate each function in $L^2(\mu)$. This question goes back to Kolmogorov and Krein and remains open for about 70 years. We show that the minimal width is stable under exponentially small perturbations of measures and may be not stable when the perturbations are only subexponentially small.

This result is applied to construct non-classical spectral measures for Sturm-Liouville operators on a finite interval. Answering a question raised by Marchenko several years ago, we show that locally there are no restrictions on spectral measures of self-adjoint extensions of Sturm-Liouville operators with potentials q in $C^\infty[0, 1)$.

This is a joint work with Alexander Borichev (Marseille).

Marcelo Viana: **Absolute continuity, Lyapunov exponents, and rigidity**

Abstract: I report on an ongoing joint project with Artur Avila and Amie Wilkinson, where ideas from the theory of linear cocycles are applied to the dynamics of partially hyperbolic diffeomorphisms. The starting point is to view the diffeomorphism as a *smooth* cocycle over itself, acting on a fiber bundle whose fibers are the center leaves. The sharpest results hold in dimension 3. We prove, in a number of situations, that absolute continuity of the foliation implies rigidity, meaning smooth conjugacy to a rigid model. Moreover, if absolute continuity breaks down then the disintegration must be atomic.

Michel Zinsmeister: **Variations of the Hausdorff dimension of quadratic Julia sets**

Abstract: We will discuss the problem of the variations of the Hausdorff dimension of Julia sets in a hyperbolic family when the polynomial converges to one with a parabolic point. A particular focus will be given on the quadratic family in the main cardioid.
