1. $2(1+2+\dots+n)$

$$= \begin{matrix} 1+2+\dots+n \\ +n+n-1+\dots+1 \end{matrix} = n\cdot(n+1)$$

2. $f'(x) = x^3+2x^2+x = x(x^2+2x+1) = x(x+1)^2$

$\Rightarrow$ $x_0=0$ och $x_-=-1$ är möjliga extrempunkter

$f''(x) = (x+1)^2 + 2x(x+1)$

$f''(0) = 1 > 0 \Rightarrow x_0 = 0$ Minimipunkt

$f''(-1) = 0$; $f'(x) < 0$ i $(-1,0) \Rightarrow x_- = -1$ Maximipunkt i $[-1,+1]$

$f'(x) > 0$ i $(0,1] \Rightarrow x_+ = 1$ (randpunkt) är Maximipunkt

3. $$\int_{-2}^{\sqrt{3}-2} \frac{dx}{(x+2)^2+3} \underset{u=x+2}{=} \int_0^{\sqrt{3}} \frac{du}{u^2+3} \underset{u=\sqrt{3}v}{=} \frac{1}{\sqrt{3}}\int_0^1 \frac{dv}{v^2+1}$$

$$= \frac{1}{\sqrt{3}} \arctan v \Big|_0^1 = \frac{1}{\sqrt{3}}\frac{\pi}{4}$$

4. $y''-6y'-7y=0 \underset{y=e^{\lambda x}}{\longrightarrow} \lambda^2-6\lambda-7 = (\lambda-7)(\lambda+1) \overset{!}{=} 0$

$\Rightarrow y_H(x) = A e^{7x} + B e^{-x}$

(→ Resonans!)

$y_p(x) = C\cdot x\cdot e^{-x}$, $y_p' = Ce^{-x}(1-x)$, $y_p'' = Ce^{-x}(-1-1+x)$

$y_p''-6y_p'-7y_p = Ce^{-x}(x-2+6x-6-7x) \overset{!}{=} e^{-x} \Rightarrow C = -\frac{1}{8}$

$\Rightarrow y(x) = Ae^{7x} + Be^{-x} - \frac{1}{8}xe^{-x}$

5. $\int_0^1 \underbrace{x^2}_{h}\underbrace{e^x}_{g'} = x^2e^x\Big|_0^1 - \int_0^1 \underbrace{2x}_{h}\underbrace{e^x}_{g} = (x^2e^x - 2xe^x)\Big|_0^1 + \int_0^1 2e^x$

$= (x^2-2x+2)e^x\Big|_0^1 = e-2 \quad (>0)$

6. $\tan x = \frac{\sin x}{\cos x} = \frac{x - x^3/6 + \dots}{1 - x^2/2 + \dots} = x\left(1 - x^2/6 + \dots\right)\left(1 + \frac{x^2}{2} + \dots\right)$

$= x\left(1 + \frac{1}{3}x^2 + \dots\right) = x + \frac{1}{3}x^3 + \dots$

7. $f'(x) := \lim_{h\to 0} \frac{f(x+h) - f(x)}{h} = \lim_{h\to 0}\left(\frac{2}{h}\left\{\begin{matrix}(x+h)^2 - 2(x+h) + 1 \\ -x^2 + 2x \quad -1\end{matrix}\right\}\right)$

$= \lim_{h\to 0} \frac{2}{h}\left\{2xh - 2h + h^2\right\} = \lim_{h\to 0}(4x - 4 + 2h) = 4x - 4$

8. $x^3 - 3x + 2 = (x-1)(x^2+x-2) = (x-1)(x-1)(x+2) = (x-1)^2(x+2)$

$\frac{1}{x^3-3x+2} \overset{!}{=} \frac{A}{x-1} + \frac{B}{(x-1)^2} + \frac{C}{(x+2)} \Rightarrow A = -\frac{1}{9},\ B = \frac{1}{3},\ C = \frac{1}{9}$

$\Rightarrow \int_3^\infty \frac{dx}{x^3-3x+2} = \frac{1}{9}\int_3^\infty dx\left(\frac{1}{x+2} - \frac{1}{x-1} + \frac{3}{(x-1)^2}\right) = \lim_{\Lambda\to\infty}\frac{1}{9}\int_3^\Lambda \dots$

$= \frac{1}{9}\lim_{\Lambda\to\infty}\left[\ln\frac{x+2}{x-1} - \frac{3}{(x-1)}\right]_3^\Lambda = 0 - \frac{1}{9}\left(\ln\frac{5}{2} - \frac{3}{2}\right) = \frac{3 - \ln\frac{25}{4}}{18} > 0$

9. $\lim_{x\to 0}\frac{(4x - \sin 4x)}{x^3}\frac{(1-(1+x)^5)}{x} = \lim_{x\to 0}\frac{((4x)^3/3! + \dots)}{x^3}\frac{(1 - [1 + 5x + \dots])}{x}$

$= \frac{64}{6}\cdot(-5) = -\frac{160}{3}$

10. $r' = \frac{-w'}{\left(w + \frac{\alpha m}{L^2}\right)^2} \Rightarrow \frac{w'^2}{(\ )^4} \overset{!}{=} \frac{2Em}{L^2}\frac{1}{(\ )^4} + \frac{2\alpha m}{L^2}\cdot\frac{1}{(\ )^3} - \frac{1}{(\ )^2}$

$\rightarrow w'^2 = \frac{2Em}{L^2} + \frac{2\alpha m}{L^2}\left(w + \frac{\alpha m}{L^2}\right) - \left(w + \frac{\alpha m}{L^2}\right)^2$

$\Leftrightarrow \quad w'^2 = \underbrace{\left(\frac{2Em}{L^2} + \frac{\alpha^2 m^2}{L^4}\right)}_{=:D^2} - w^2$

$w = D\cos(\varphi + \varphi_0)$

$w' = D\cdot(-\sin(\varphi+\varphi_0)) \Rightarrow \quad w'^2 + w^2 = D^2\left(\sin^2(\varphi+\varphi_0) + \cos^2(\varphi+\varphi_0)\right) = D^2$