FÖRSLAG TILL LÖSNING TENTAMEN MATEMATIK BASKURS SF1612

MATEMATIK I, DEL A SF1623. 2008-01-19

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1. $x^4 + x^3 + x^2 + x < 0$

$x(x^3 + x^2 + x + 1) < 0$

$x(x^3 + x + x^2 + 1) < 0$

$x(x(x^2+1) + x^2 + 1) < 0$

$x(x^2+1)(x+1) < 0$

x		-1		0		
x	$-$		$-$	0	$+$	$+$
x^2+1	$+$		$+$		$+$	$+$
$x+1$	$-$	0	$+$		$+$	$+$
$x(x^2+1)(x+1)$	$+$	0	$-$	0	$+$	$+$

$\therefore$ $x(x^2+1)(x+1) < 0$ DÅ $-1 < x < 0$

<u>SVAR:</u> $-1 < x < 0$

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2. $\ln x = t$

a) $\ln x^3 = 3\ln x = 3t$

b) $\ln \sqrt[5]{x^2} = \ln x^{\frac{2}{5}} = \frac{2}{5}\ln x = \frac{2}{5}t$

c) $\ln \frac{1}{x} = \ln 1 - \ln x = 0 - t = -t$

d) $\ln \frac{\sqrt[3]{x}}{\sqrt{x}} = \ln \frac{x^{\frac{1}{3}}}{x^{\frac{1}{2}}} = \ln x^{\frac{1}{3}-\frac{1}{2}} = \ln x^{-\frac{1}{6}} = -\frac{1}{6}\ln x = -\frac{1}{6}t$

<u>SVAR:</u> a) $3t$ b) $\frac{2}{5}t$ c) $-t$ d) $-\frac{1}{6}t$

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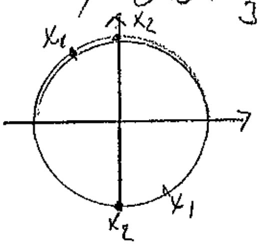
3. $\sin\left(2x + \frac{\pi}{3}\right) = -\frac{\sqrt{3}}{2}$

1) $2x + \frac{\pi}{3} = -\frac{\pi}{3} + n\cdot 2\pi \iff 2x = -\frac{2\pi}{3} + n\cdot 2\pi \iff x = -\frac{\pi}{3} + n\cdot\pi$

2) $2x + \frac{\pi}{3} = \pi + \frac{\pi}{3} + n\cdot 2\pi \iff 2x = \pi + n\cdot 2\pi \iff x = \frac{\pi}{2} + n\cdot\pi$



1) $\cos\left(-\frac{\pi}{3}\right) = \frac{1}{2}$, $\cos\frac{2\pi}{3} = -\frac{1}{2}$

2) $\cos\frac{\pi}{2} = 0$, $\cos\left(-\frac{\pi}{2}\right) = 0$

<u>SVAR:</u> $\cos x$ KAN VARA 0 ELLER $\pm\frac{1}{2}$

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4. $$\sum_{k=1}^{19}(2^k-4k+8) = \sum_{k=1}^{19} 2^k + \sum_{k=1}^{19}(8-4k) = S_1 + S_2$$

$$S_1 = \sum_{k=1}^{19} 2^k = \left| \begin{matrix}\text{GEOM.}\\ \text{TALFÖLJD}\\ k=2\end{matrix} \right| = 2\left(\frac{2^{19}-1}{2-1}\right) = 2(2^{19}-1) = 2^{20}-2$$

$$S_2 = \sum_{k=1}^{19}(8-4k) = \left| \begin{matrix}\text{ARITM. TALF.}\\ d=-4,\ n=19\\ a_{19}=4+19(-4)\\ =-68\end{matrix} \right| = 19\cdot\frac{(4-68)}{2} = 19\cdot\frac{(-64)}{2} = 19\cdot -32$$

$$S_1+S_2 = 2^{20}-2+19\cdot -32 = 2^{20}-610.$$

SVAR: $2^{20}-610$.

5. $$\frac{(1+i)^9}{(1-i)^7} = \frac{(1+i)^7}{(1-i)^7}\cdot(1+i)^2 = \left(\frac{(1+i)(1+i)}{(1-i)(1+i)}\right)^7\cdot(1+i)^2 =$$

$$= \left(\frac{(1+i)^2}{2}\right)^7\cdot(1+i)^2 = \frac{((1+i)^2)^8}{2^7} = \frac{(1+2i-1)^8}{2^7} = \frac{2^8\cdot(i)^8}{2^7} =$$

$$= 2\cdot(i^2)^4 = 2\cdot(-1)^4 = 2$$

SVAR: 2

6. $$f(x) = \sqrt{9+x} - 3$$

a) D_f: $9+x \geq 0 \iff x \geq -9$

V_f: $f(x) \geq -3$

b) $$y = \sqrt{9+x} - 3$$

$$y+3 = \sqrt{9+x}$$

$$(y+3)^2 = 9+x$$

$$x = (y+3)^2 - 9 \qquad D_{f^{-1}}: x \geq -3$$

$$f^{-1}(x) = (x+3)^2 - 9. \qquad V_{f^{-1}}: f^{-1}(x) \geq -9$$

c) $$f(f^{-1}(x)) = \sqrt{9+(x+3)^2-9} - 3 = \sqrt{(x+3)^2} - 3 = |x+3| - 3$$

$$= x+3-3 = x$$

SVAR: a) D_f: $x \geq -9$, V_f: $f(x) \geq -3$

b) $f^{-1}(x) = (x+3)^2 - 9$

c) x.

7. $|x-1| + 2|x-2| = a$

$$|x-1| = \begin{cases} x-1 & x \geq 1 \\ -(x-1) & x < 1 \end{cases} \qquad |x-2| = \begin{cases} x-2 & x \geq 2 \\ -(x-2) & x < 2 \end{cases}$$

x: I | 1 | II | 2 | III

I: $x < 1$ $\quad -(x-1) - 2(x-2) = a$
$$-3x + 5 = a$$
$$x = \frac{5-a}{3}$$

$x<1$ GER:
$$\frac{5-a}{3} < 1$$
$$5-a < 3$$
$$5-3 < a$$
$$a > 2.$$

II $1 \leq x < 2$ $\quad x-1-2(x-2) = a$
$$-x+3 = a$$
$$x = 3-a$$

$1 \leq x < 2$ GER:
$$1 \leq 3-a < 2$$
$$-2 \leq -a < -1$$
$$1 < a \leq 2$$

III $x \geq 2$ $\quad x-1+2(x-2) = a$
$$3x-5 = a$$
$$x = \frac{a+5}{3}$$

$x \geq 2$ GER:
$$\frac{a+5}{3} \geq 2$$
$$a \geq 6-5$$
$$a \geq 1$$

a: 1, 2

SVAR:
- $a<1$ GER INGEN LÖSNING
- $a=1$ GER EN LÖSNING (DVS $x = \frac{a+5}{3}$)
- $1 < a \leq 2$ GER TVÅ LÖSNINGAR (DVS $x = \frac{a+5}{3}$, $x = 3-a$)
- $a > 2$ GER TVÅ LÖSNINGAR (DVS $x = \frac{5-a}{3}$, $x = \frac{a+5}{3}$)

8. $n \geq 2$, $1 \leq k \leq n-1$

VISA $\binom{n-1}{k-1} + \binom{n-1}{k} = \binom{n}{k}$

$$\binom{n-1}{k-1} + \binom{n-1}{k} = \frac{(n-1)!}{(n-1-k+1)!\,(k-1)!} + \frac{(n-1)!}{(n-k-1)!\,k!} =$$

$$\frac{(n-1)!}{(n-k)!\,(k-1)!} + \frac{(n-1)!}{(n-k-1)!\,k!} = \frac{(n-1)!\,(k!\,(n-k-1)! + (n-k)!\,(k-1)!)}{(n-k)!\,(n-k-1)!\,k!\,(k-1)!} =$$

$k! = k(k-1)!$, $(n-k)! = (n-k)(n-k-1)!$

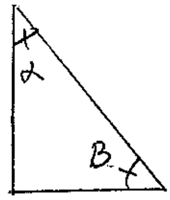
$$= \frac{(n-1)!\,(k-1)!\,(n-k-1)!\,(k+n-k)}{(n-k)!\,(n-k-1)!\,k!\,(k-1)!} = \frac{(n-1)!\cdot n}{(n-k)!\,k!} = \frac{n!}{(n-k)!\,k!} = \binom{n}{k}$$

VSV.

(ALT. BEVIS SE BOKEN SID 64)

9. $f(x) = \arctan x + \operatorname{arccot} x$ ÄR KONSTANT.

PÅST! $\arctan x + \operatorname{arccot} x = \frac{\pi}{2}$



$\alpha + \beta = \frac{\pi}{2}$

VISA ATT $\arctan x = \frac{\pi}{2} - \operatorname{arccot} x$

BEVIS! $-\frac{\pi}{2} \leq \frac{\pi}{2} - \operatorname{arccot} x \leq \frac{\pi}{2}$

SÅ $\tan\left(\frac{\pi}{2} - \operatorname{arccot} x\right) = \cot(\operatorname{arccot} x) = x$

OCH $\tan(\arctan x) = x$

FÖLJER ATT $\frac{\pi}{2} - \operatorname{arccot} x = \arctan x$

OCH $\arctan x + \operatorname{arccot} x = \frac{\pi}{2}$.

(Alt. se boken s. 121)