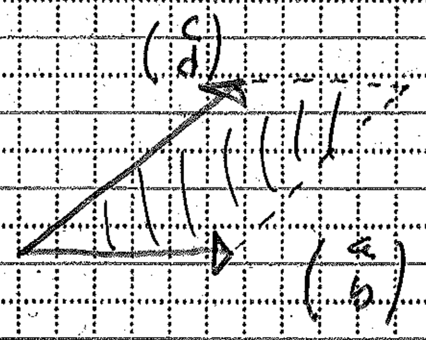
$$A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$$

Def $\det A = \begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - bc$

Tolkning:



$\sim$ Area $= |ad - bc|$

Sats I

(i) $\det A^t = \det A$

dvs $\begin{vmatrix} a & b \\ c & d \end{vmatrix} = \begin{vmatrix} a & c \\ b & d \end{vmatrix}$

Motsv. gäller även kolumner p.g.a. (i)

(ii) $\begin{vmatrix} \lambda a & \lambda b \\ c & d \end{vmatrix} = \lambda \begin{vmatrix} a & b \\ c & d \end{vmatrix}$

(iii) $\begin{vmatrix} a & b \\ c+ka & d+kb \end{vmatrix} = \begin{vmatrix} a & b \\ c & d \end{vmatrix}$

(iv) $\begin{vmatrix} a & b \\ c & d \end{vmatrix} = - \begin{vmatrix} c & d \\ a & b \end{vmatrix}$

(v) $\begin{vmatrix} a & b \\ 0 & 0 \end{vmatrix} = \begin{vmatrix} 0 & 0 \\ a & b \end{vmatrix} = 0$

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$$A = \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix}$$

<u>Def</u>: $\det A =$

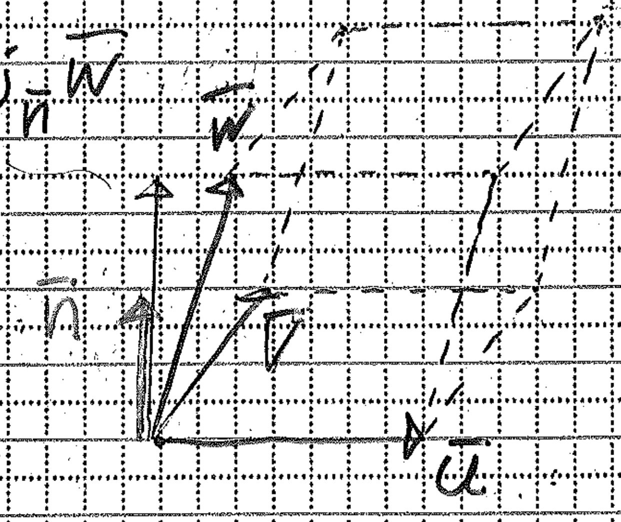
$$= a_{11}\begin{vmatrix} a_{22} & a_{23} \\ a_{32} & a_{33} \end{vmatrix} - a_{12}\begin{vmatrix} a_{21} & a_{23} \\ a_{31} & a_{33} \end{vmatrix} + a_{13}\begin{vmatrix} a_{21} & a_{22} \\ a_{31} & a_{32} \end{vmatrix}$$

<u>Def</u>: $\det \begin{pmatrix} a_{11} & \cdots & a_{1n} \\ & & \\ a_{n1} & \cdots & a_{nn} \end{pmatrix} =$

$$= a_{11}\left|\ \cancel{a_{11}}\ \right| - a_{12}\left|\ \cancel{a_{12}}\ \right| + \dots + (-1)^{n} a_{1n}\left|\ \cancel{a_{1n}}\ \right|$$

SATS I (i) – (v) gäller
även för $n \times n$ determinanter

$\text{proj}_{\bar{n}}\bar{w}$



Parallellepipedens volym

trippelskalärprodukt och 3×3 determinanten

$$\text{Vol} = \text{basyta} \cdot \text{höjd}$$

$$\text{basyta} = |\bar{u} \times \bar{v}|$$

$$h = \text{höjd} = |\text{proj}_{\bar{n}}\bar{w}|$$

$$\bar{n} = \bar{u} \times \bar{v}$$

$$\text{proj}_{\bar{n}}\bar{w} = \frac{\bar{n} \cdot \bar{w}}{|\bar{n}|^2}\bar{n} = \frac{(\bar{u} \times \bar{v}) \cdot \bar{w}}{|\bar{u} \times \bar{v}|^2}(\bar{u} \times \bar{v})$$

$$h = \text{höjd} = \left| \frac{(\bar{u} \times \bar{v}) \cdot \bar{w}}{|\bar{u} \times \bar{v}|^2}(\bar{u} \times \bar{v}) \right| = \frac{|(\bar{u} \times \bar{v}) \cdot \bar{w}|}{|\bar{u} \times \bar{v}|^2}|\bar{u} \times \bar{v}|$$

$$= \frac{|(\bar{u} \times \bar{v}) \cdot \bar{w}|}{|\bar{u} \times \bar{v}|}$$

$$\boxed{\text{Vol} = \text{basyta} \cdot \text{höjd} = \cancel{|\bar{u} \times \bar{v}|} \cdot \frac{|(\bar{u} \times \bar{v}) \cdot \bar{w}|}{\cancel{|\bar{u} \times \bar{v}|}} = |(\bar{u} \times \bar{v}) \cdot \bar{w}|}$$

Om $\bar{u} = \begin{pmatrix} u_1 \\ u_2 \\ u_3 \end{pmatrix}$, $\bar{v} = \begin{pmatrix} v_1 \\ v_2 \\ v_3 \end{pmatrix}$, $\bar{w} = \begin{pmatrix} w_1 \\ w_2 \\ w_3 \end{pmatrix}$

är trippelskalärprodukten

$$\boxed{(\bar{u} \times \bar{v}) \cdot \bar{w}} = \left(\begin{pmatrix} u_1 \\ u_2 \\ u_3 \end{pmatrix} \times \begin{pmatrix} v_1 \\ v_2 \\ v_3 \end{pmatrix} \right) \cdot \begin{pmatrix} w_1 \\ w_2 \\ w_3 \end{pmatrix}$$

$$= \begin{pmatrix} u_2 v_3 - u_3 v_2 \\ u_3 v_1 - u_1 v_3 \\ u_1 v_2 - u_2 v_1 \end{pmatrix} \cdot \begin{pmatrix} w_1 \\ w_2 \\ w_3 \end{pmatrix}$$

$$= w_1 (u_2 v_3 - u_3 v_2) - w_2 (u_1 v_3 - u_3 v_1) + w_3 (u_1 v_2 - u_2 v_1)$$

$$= w_1 \begin{vmatrix} u_2 & u_3 \\ v_2 & v_3 \end{vmatrix} - w_2 \begin{vmatrix} u_1 & u_3 \\ v_1 & v_3 \end{vmatrix} + w_3 \begin{vmatrix} u_1 & u_2 \\ v_1 & v_2 \end{vmatrix}$$

$$= \boxed{\begin{vmatrix} u_1 & u_2 & u_3 \\ v_1 & v_2 & v_3 \\ w_1 & w_2 & w_3 \end{vmatrix}}$$

OBS:

$$\begin{pmatrix} u_1 \\ u_2 \\ u_3 \end{pmatrix} \times \begin{pmatrix} v_1 \\ v_2 \\ v_3 \end{pmatrix} = \begin{pmatrix} \begin{vmatrix} u_2 & v_2 \\ u_3 & v_3 \end{vmatrix} \\ -\begin{vmatrix} u_1 & v_1 \\ u_3 & v_3 \end{vmatrix} \\ \begin{vmatrix} u_1 & v_1 \\ u_2 & v_2 \end{vmatrix} \end{pmatrix} =$$

$$= \begin{vmatrix} u_2 & v_2 \\ u_3 & v_3 \end{vmatrix} \bar{e}_x - \begin{vmatrix} u_1 & v_1 \\ u_3 & v_3 \end{vmatrix} \bar{e}_y + \begin{vmatrix} u_1 & v_1 \\ u_2 & v_2 \end{vmatrix} \bar{e}_z$$

$$= \begin{vmatrix} \bar{e}_x & u_1 & v_1 \\ \bar{e}_y & u_2 & v_2 \\ \bar{e}_z & u_3 & v_3 \end{vmatrix}$$

$$\left(= \begin{vmatrix} \bar{e}_x & \bar{e}_y & \bar{e}_z \\ u_1 & u_2 & u_3 \\ v_1 & v_2 & v_3 \end{vmatrix} \right)$$