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# Lösningsförslag till kontrollskrivning 3B i 5B1147 Envariabelanalys för E, ht 2006.

1.
$$\int \frac{3x-1}{x^2-2x-3}\,dx = ?$$

$$x^2-2x-3=0 \iff x = 1 \pm \sqrt{1+3} = 1 \pm 2 = \begin{cases} 3 \\ -1 \end{cases}$$
$$\implies x^2-2x-3 = (x+1)(x-3).$$

Så
$$\int \frac{3x-1}{x^2-2x-3}\,dx = \int \frac{3x-1}{(x+1)(x-3)}\,dx = \{\text{handpåläggning}\}$$
$$= \int \left(\frac{1}{x+1} + \frac{2}{x-3}\right) dx = \ln|x+1| + 2\ln|x-3| + C.$$

2.
$$\int_0^1 \frac{dx}{x^2+3} = \frac{1}{3}\int_0^1 \frac{dx}{1+(x/\sqrt{3})^2} = \frac{1}{3}\left[\sqrt{3}\arctan\frac{x}{\sqrt{3}}\right]_0^1$$
$$= \frac{1}{\sqrt{3}}\arctan\frac{1}{\sqrt{3}} - 0 = \frac{\pi}{6\sqrt{3}}.$$

3.
$$\int_0^{\pi/2} (\cos x)^3\,dx = \int_0^{\pi/2} (\cos x)^2 \cdot \cos x\,dx = \int_0^{\pi/2} (1-(\sin x)^2)\cdot\cos x\,dx$$
$$= \{u = \sin x,\ du = \cos x\,dx\} = \int_0^1 (1-u^2)\,du$$
$$= \left[u - \frac{u^3}{3}\right]_0^1 = \frac{2}{3}.$$