10.27.

$$\mathbf{F} = (x, y, (z-1)^2)$$

$$S: \; x^2 + y^2 + z^2 = 1, \; z \geq 0.$$

S är ej sluten. Slut med ytan S_1.

$$S_1: \; x^2 + y^2 = 1, \; z = 0.$$

$$div\mathbf{F} = 1 + 1 + 2(z-1) = 2z$$

$$\iint_{S\ S_1} \mathbf{F}\cdot\hat{\mathbf{n}}d\sigma = \iiint_K 2zdxdydz$$

$$\iiint_K 2zdxdydz = \iint_{D_{xy}} \left[\int_{z=0}^{\sqrt{1-x^2-y^2}} 2zdz\right]dxdy$$

$$\iiint_K 2zdxdydz = \iint_{D_{xy}} \left\{1-x^2-y^2\right\}dxdy$$

$$\iiint_K 2zdxdydz = \iint_{D_{r\theta}} \left\{1-r^2\right\}rdrd\theta$$

$$\iiint_K 2z\,dx\,dy\,dz = 2\pi\left(\frac{1}{2} - \frac{1}{4}\right) = \frac{\pi}{2}$$

$$\iint_S \mathbf{F}\cdot\hat{\mathbf{n}}\,d\sigma + \iint_{S_1} \mathbf{F}\cdot\hat{\mathbf{n}}_1\,d\sigma = \frac{\pi}{2}$$

$$\iint_{S_1} \mathbf{F}\cdot\hat{\mathbf{n}}_1\,d\sigma = \iint_{S_1} (x,y,1)\cdot(0,0,-1)\,d\sigma = -\pi$$

SVAR:

$$\text{Flödet } \Phi = \iint_S \mathbf{F}\cdot\hat{\mathbf{n}}\,d\sigma = \frac{3\pi}{2}.$$