

6.19.

$$\int_D f(x, y) dx dy = \int_{D_{uv}} f(x(u, v), y(u, v)) \left| \frac{d(x, y)}{d(u, v)} \right| du dv$$

Sätt:

$$u = x + y$$

$$v = x - y$$

Funktionaldeterminanten

$$\frac{d(u, v)}{d(x, y)} = \begin{vmatrix} 1 & 1 \\ 1 & -1 \end{vmatrix} = -2$$

$$\left| \frac{d(x, y)}{d(u, v)} \right| = \frac{1}{|-2|} = \frac{1}{2}$$

$$\int_D (x^2 - y^2)^{10} dx dy = \int_{D_{uv}} (uv)^{10} \frac{1}{2} du dv$$

$$\int_{D_{uv}} (uv)^{10} \frac{1}{2} du dv = \frac{1}{2} \int_{u=-1}^1 u^{10} du \int_{v=-1}^1 v^{10} dv$$

SVAR:

$$\int_D (x^2 - y^2)^{10} dx dy = \frac{1}{2} \cdot 2 \cdot 2 \cdot \frac{1}{11} \cdot \frac{1}{11} = \frac{2}{121}$$