

6.22.

Sätt:

$$\begin{array}{l} x = r \cos \theta \\ y = r \sin \theta \end{array} \quad D_{r\theta} : \quad \begin{array}{l} r : 0 \quad 5 \\ \theta : \frac{\pi}{4} \quad \frac{3\pi}{4} \end{array} \quad dx dy = r dr d\theta$$

$$V = \int_D x^2 e^{x^2 + y^2} dx dy = \int_{D_{r\theta}} r^2 \cos^2 \theta e^{r^2} r dr d\theta$$

$$V = \frac{1}{2} \int_{\theta = \frac{\pi}{4}}^{\frac{3\pi}{4}} (1 + \cos 2\theta) d\theta \int_{r=0}^5 r^2 e^{r^2} r dr$$

$$\int_{r=0}^5 r^2 e^{r^2} r dr = \int_{t=0}^{t=25} \frac{1}{2} t e^t dt$$

$t = r^2 \quad r = 5 \quad t = 25$
 $dt = 2r dr \quad r = 0 \quad t = 0$

$$V = \frac{1}{2} \left[\frac{3\pi}{4} - \frac{\pi}{4} + \frac{1}{2} \sin 2 \left(\frac{3\pi}{4} - \sin 2 \frac{\pi}{4} \right) \right] \frac{1}{2} (25e^{25} - e^{25} - 0 + e^0)$$

$$V = \frac{1}{2} \left[\frac{\pi}{2} - 1 + \frac{1}{2} (24e^{25} + 1) \right] = \frac{\pi - 2}{8} (24e^{25} + 1)$$

SVAR: $\int_D x^2 e^{x^2 + y^2} dx dy = \frac{\pi - 2}{8} (24e^{25} + 1)$