

7.16.

$$\text{Molnets totala massa ges av: } M = \int_{R^3} e^{-(x^2 + 2y^2 + 3z^2)} dx dy dz.$$

Inför nya koordinater :

$$x = r \sin \theta \cos \varphi$$

$$y\sqrt{2} = r \sin \theta \sin \varphi \quad dx dy dz = \frac{1}{\sqrt{2}} \frac{1}{\sqrt{3}} r^2 \sin \theta dr d\theta d\varphi$$

$$z\sqrt{3} = r \cos \theta$$

$$D_{r\theta\varphi} = \left\{ (r, \theta, \varphi) : 0 < r < \infty, 0 < \theta < \pi, 0 < \varphi < 2\pi \right\}$$

$$M = \int_{D_{r\theta\varphi}} e^{-r^2} \frac{1}{\sqrt{6}} r^2 \sin\theta dr d\theta d\varphi = \frac{2\pi}{\sqrt{6}} \int_{r=0}^{\infty} e^{-r^2} r^2 dr$$

$$e^{-r^2} r^2 dr = r \frac{e^{-r^2}}{-2} - \frac{e^{-r^2}}{-2} \cdot 1 dr$$

$$M = \frac{2\pi}{\sqrt{6}} \int_0^{\infty} e^{-r^2} r^2 dr = \frac{2\pi}{\sqrt{6}} \left[\frac{1}{2} \sqrt{\pi} \right] = \sqrt{\frac{\pi^3}{6}}$$

SVAR:

Molnets totala massa ges av : $M = \sqrt{\frac{\pi^3}{6}}$.