

7.4.

$$M = \int_{D_{xy}} \int_{z=\sqrt{x^2+y^2}}^{\sqrt{1-x^2-y^2}} \frac{z}{1+x^2+y^2} dz \, dxdy = \int_{D_{xy}} \frac{1-x^2-y^2-(x^2+y^2)}{2(1+x^2+y^2)} \, dxdy$$

$$D_{xy} = (x, y) : x^2 + y^2 \leq \frac{1}{2}$$

Sätt:

$$\begin{array}{l} x = r \cos \theta \\ y = r \sin \theta \end{array} \quad D_{r\theta} : \begin{array}{l} r : 0 \\ \theta : 0 \end{array} \quad \frac{1}{\sqrt{2}} \quad dxdy = r dr d\theta$$

$$M = \int_{D_{r\theta}} \frac{1-2r^2}{2(1+r^2)} \, r dr d\theta$$

$$M = \frac{1}{2} \cdot 2\pi \int_{r=0}^{\frac{1}{\sqrt{2}}} \frac{1-2r^2}{1+r^2} r dr$$

$$\begin{aligned} t &= 1+r^2 & r &= \frac{1}{\sqrt{2}} & t &= \frac{3}{2} \\ dt &= 2r dr & r &= 0 & t &= 1 \end{aligned}$$

$$M = \pi \int_{t=1}^{\frac{3}{2}} \frac{3-2t}{2t} dt = \frac{\pi}{2} \int_{t=1}^{\frac{3}{2}} \left(\frac{3}{t} - 2 \right) dt$$

SVAR:

$$\int_D \frac{z}{1+x^2+y^2} dx dy dz = \frac{\pi}{2} \left(3 \ln \frac{3}{2} - 1 \right)$$