8.16.

a)

$$\begin{cases} x = 2\sin\theta\cos\varphi \\ y = 2\sin\theta\sin\varphi \\ z = 2\cos\theta \end{cases}$$

$$D_{\theta\varphi}: \begin{cases} \varphi : 0 \to 2\pi \\ \theta : 0 \to \dfrac{\pi}{3} \end{cases}$$

$$\mathbf{r} = \left(2\sin\theta\cos\varphi,\ 2\sin\theta\sin\varphi,\ 2\cos\theta\right)$$

b)

$$\dot{\mathbf{r}}_\theta = (2\cos\theta\cos\varphi,\ 2\cos\theta\sin\varphi,\ -2\sin\theta)$$

$$\dot{\mathbf{r}}_\varphi = (-2\sin\theta\sin\varphi,\ 2\sin\theta\cos\varphi,\ 0)$$

$$\mathbf{n} = \dot{\mathbf{r}}_\theta \times \dot{\mathbf{r}}_\varphi$$

$$\mathbf{n} = \begin{vmatrix} \mathbf{e}_1 & \mathbf{e}_2 & \mathbf{e}_3 \\ 2\cos\theta\cos\varphi & 2\cos\theta\sin\varphi & -2\sin\theta \\ -2\sin\theta\sin\varphi & 2\sin\theta\cos\varphi & 0 \end{vmatrix}$$

$$\mathbf{n} = (4\sin^2\theta\cos\varphi,$$
$$-(-4\sin^2\theta\sin\varphi),$$
$$4\sin\theta\cos\theta\cos^2\varphi + 4\sin\theta\cos\theta\sin^2\varphi)$$

$$\mathbf{n} = \left(4\sin^2\theta\cos\varphi, 4\sin^2\theta\sin\varphi,\ 4\sin\theta\cos\theta\right)$$

$$\mathbf{n} = 2\sin\theta\left(2\sin\theta\cos\varphi, 2\sin\theta\sin\varphi, 2\cos\theta\right)$$

$$\mathbf{e}_n = \frac{\mathbf{n}}{|\mathbf{n}|} = \frac{\mathbf{r}}{|\mathbf{r}|} = \frac{\mathbf{r}}{2}$$

c)

$$A = \iint\limits_{D_{\theta\varphi}} |\mathbf{n}|\, d\theta\, d\varphi = \iint\limits_{D_{\theta\varphi}} |2\ \ 2\sin\theta|\, d\theta\, d\varphi$$

$$A = \iint\limits_{D_{\theta\varphi}} 2\ \ 2\sin\theta\, d\theta\, d\varphi = 4\ \ 2\pi\ \ \left(-\cos\frac{\pi}{3} + \cos 0\right) = 4\pi$$

d)

$$A = \iint_{D1_{\theta\varphi}} R \;\; R\sin\theta \, d\theta \, d\varphi = R^2 \;\; 2\pi \;\; (-\cos\theta_0 + \cos 0)$$

$$A = R^2 \;\; 2\pi \;\; (-\frac{h}{R} + 1) = 2\pi R(R - h)$$

Alternativ parameterframställning

$$\mathbf{r} = \left(x,\ y,\ \sqrt{4 - x^2 - y^2}\right),\quad x^2 + y^2 \le 3$$

$$\hat{\mathbf{n}} = \frac{\mathbf{r}}{2},\quad \cos\gamma = \frac{z}{2} = \frac{\sqrt{4 - x^2 - y^2}}{2}$$

$$\text{Arean } A = \iint_S dS = \iint_{D_{xy}} \frac{dxdy}{|\cos\gamma|} = \iint_{D_{xy}} \frac{2\,dxdy}{\sqrt{16 - x^2 - y^2}}$$

$$A = \iint_{D_{r\theta}} \frac{2\,rdrd\theta}{\sqrt{4 - r^2}} = 2\ \ 2\pi \int_{r=0}^{\sqrt{3}} \frac{rdr}{\sqrt{4 - r^2}} = 4\pi(-1 + 2) = 4\pi$$