9.33.

$$2xydx-(y^2+3a^2-3x^2)dy$$

$$dU(x,y)=g(y)\ \ 2xydx-g(y)\ \ (y^2+3a^2-3x^2)dy$$

$$dU(x,y)=P(x,y)dx+Q(x,y)dy$$

$$\begin{cases}\dfrac{\partial P(x,y)}{\partial y}=g'(y)\ \ 2xy+g(y)\ \ 2x\\ \dfrac{\partial Q(x,y)}{\partial x}=-g(y)\ \ (-6x)\end{cases}$$

$$\frac{\partial P(x,y)}{\partial y} = \frac{\partial Q(x,y)}{\partial x} \Leftrightarrow g'(y)\ \ 2xy + g(y)\ \ 2x = g(y)\ \ 6x$$

$$yg'(y) - 2g(y) = 0$$

$$y^{-2}g'(y) - 2y^{-3}g(y) = 0$$

$$\frac{d}{dy}\{y^{-2}g(y)\} = 0$$

$$g(y) = C_1 y^2$$

Välj $g(y) = y^2$.

$$dU(x,y) = 2xy^3dx - (y^4 + 3a^2y^2 - 3x^2y^2)dy$$

$$\begin{cases} U'_x(x,y) = 2xy^3 & U(x,y) = x^2y^3 + g(y) \\ U'_y(x,y) = -y^4 - 3a^2y^2 + 3x^2y^2 & U'_y(x,y) = 3x^2y^2 + g'(y) \end{cases}$$

$$g'(y) = -y^4 - 3a^2y^2 \ , \ g(y) = -\frac{y^5}{5} - a^2y^3 + C$$

SVAR:

$$U(x,y) = x^2y^3 - \frac{y^5}{5} - a^2y^3 + C$$