9.51.

$$\int_{\gamma} \frac{(x-y)dx + (x+y)dy}{x^2+y^2}$$

$$\begin{cases} P(x,y) = \dfrac{x-y}{x^2+y^2} \\ Q(x,y) = \dfrac{x+y}{x^2+y^2} \end{cases}$$

$$
\begin{cases}
\dfrac{\partial P(x,y)}{\partial y} = \dfrac{(x^2+y^2)\,(-1)-(x-y)\,2y}{(x^2+y^2)^2} \\[2ex]
\dfrac{\partial Q(x,y)}{\partial x} = \dfrac{(x^2+y^2)\,1-(x+y)\,2x}{(x^2+y^2)^2}
\end{cases}
$$

$$
\begin{cases}
\dfrac{\partial P(x,y)}{\partial y} = \dfrac{y^2-x^2-2xy}{(x^2+y^2)^2} \\[2ex]
\dfrac{\partial Q(x,y)}{\partial x} = \dfrac{y^2-x^2-y2x}{(x^2+y^2)^2}
\end{cases}
$$

Origo är en singulär punkt.

Byt väg !

Tag : $\begin{cases} x = \varepsilon \cos t \\ y = \varepsilon \sin t \end{cases} \begin{cases} dx = -\varepsilon \sin t\, dt \\ dy = \varepsilon \cos t\, dt \end{cases} \quad t : 0 \to 4\pi .$

$$\int_\gamma \frac{(x-y)dx + (x+y)dy}{x^2+y^2} =$$

$$= \int_{t=0}^{4\pi} \frac{\varepsilon(\cos t - \sin t)(-\varepsilon \sin t) + \varepsilon(\cos t + \sin t)\varepsilon \cos t}{\varepsilon^2} dt$$

$$\int_\gamma \frac{(x-y)dx + (x+y)dy}{x^2+y^2} = \int_{t=0}^{4\pi} dt = 4\pi$$

SVAR:

$$\boxed{\int_\gamma \frac{(x-y)dx + (x+y)dy}{x^2+y^2} = 4\pi}$$