

Lösningsskisser kap 9.4: Naturlig deduktion i modallogik

9.4.2 $\Box B$
 $\left[\begin{array}{l} A \\ B \end{array} \right. \text{ (}\Box E\text{)}$
 $A \rightarrow B$
 $\Box(A \rightarrow B)$

9.4.3 $\Box \neg A$
 $\left[\begin{array}{l} A \\ \neg B \\ \neg A \end{array} \right. \text{ (}\Box E\text{)}$
 $\neg B$
 $A \rightarrow B$
 $\Box(A \rightarrow B)$

9.4.4 $\Diamond(A \& \Diamond B)$
 $\left[\begin{array}{l} A \& \Diamond B \\ A \\ \Diamond B \\ \Diamond A \end{array} \right.$
 $\Diamond(A \& \Diamond B)$
 $\Diamond A \& \Diamond B \text{ (}\Diamond E\text{)}$

9.4.5 $\left[\begin{array}{l} \Diamond A \\ \Box \Box A \\ \Diamond A \rightarrow \Box \Box A \\ \Box(\dots) \\ \Diamond \Box(\dots) \end{array} \right.$

9.4.7 $\Box A$
 $\left[\begin{array}{l} \Diamond \neg A \\ \neg B \\ \neg A \\ A \end{array} \right. \text{ (}\Box E\text{)}$
 $\neg A$
 $\neg \neg B$
 B
 $\Diamond \neg A \rightarrow B$

9.4.8 $\Diamond A \vee \Diamond B$
 $\left[\begin{array}{l} \Diamond A \\ A \\ A \vee B \\ \Diamond(A \vee B) \end{array} \right. \text{ (}\Diamond E\text{)}$
 $\Diamond(A \vee B)$
 $\left[\begin{array}{l} \Diamond B \\ \vdots \\ \Diamond(A \vee B) \end{array} \right.$
 $\Diamond(A \vee B) \text{ (}\vee E\text{)}$

9.4.9 $\Box(\Diamond A \vee \Diamond B)$
 $\Diamond A \vee \Diamond B$
 $\left[\begin{array}{l} \Diamond A \\ \Box \Box A \text{ (}\Box I\text{)}$
 $\Box \Box A \vee \Box \Box B$
 $\left[\begin{array}{l} \Diamond B \\ \Box \Box B \\ \Box \Box A \vee \Box \Box B \end{array} \right.$
 $\Box \Box A \vee \Box \Box B \text{ (}\vee E\text{)}$

9.4.10 $\Box(A \rightarrow \neg)$
 $A \rightarrow \neg$
 $\left[\begin{array}{l} \Diamond A \\ A \\ \neg \end{array} \right. \text{ (}\Diamond E\text{)}$
 \neg
 $\neg \Diamond A$

9.4.11 först \vdash
 $\neg \Box A$
 $\left[\begin{array}{l} \Diamond \neg A \\ \neg A \\ \neg \neg A \\ A \\ \Box A \\ \neg \end{array} \right.$
 $\neg \neg \neg A$
 $\Diamond \neg A$
 $\text{sedan } \vdash$

9.4.12 (med maximal formel)
 A
 $\Diamond A$
 $\left[\begin{array}{l} \Box \Box A \text{ (}\Box I\text{)}$
 $\Diamond A \rightarrow \Box \Box A$
 $\Box \Box A \text{ (}\rightarrow E\text{)}$

9.4.13 $A \rightarrow \Box B$
 $\Box(B \rightarrow C)$
 $\left[\begin{array}{l} \Box B \\ B \\ B \rightarrow C \\ C \\ \Box C \\ \Box B \rightarrow \Box C \\ A \\ \Box B \\ \Box C \text{ (}\rightarrow E\text{)}$
 $A \rightarrow \Box C$

9.4.14 $\Diamond \Box A$
 $\Box A$
 $\Box A \text{ (}\Diamond E\text{)}$
 A

9.4.15 $\left[\begin{array}{l} \Diamond \Box A \\ \Box A \\ A \\ \Diamond A \\ \Box \Box A \text{ (}\Diamond E\text{)}$
 $\Diamond \Box A \rightarrow \Box \Box A$

9.4.16 $\neg(\Box(A \rightarrow \Box B) \vee \Box(\Box B \rightarrow \Box A))$
 $\left[\begin{array}{l} \Box A \rightarrow \Box B \\ \Box(\Box A \rightarrow \Box B) \text{ (}\Box I\text{)}$
 $\Box(\Box A \rightarrow \Box B) \vee \Box(\Box B \rightarrow \Box A)$
 $\neg(\Box A \rightarrow \Box B)$
 $\Box B \rightarrow \Box A$
 \vdots (Samma sätt som ovan)
 $\neg(\Box B \rightarrow \Box A)$
 $\Box A$

9.4.17 $\Box \Box A \rightarrow \Box \Box A$
 $\left[\begin{array}{l} \Box A \\ \Box \Box A \text{ (}\Box I\text{)}$
 $\Diamond \Box A \text{ (}\rightarrow E\text{)}$
 $\left[\begin{array}{l} \Box A \\ \Box A \text{ (}\Diamond E\text{)}$
 $\Diamond A \rightarrow \Box A$

9.4.18 samma 6 första rader som

9.4.20 $\Box(\Box A \vee \Box B)$
 $\neg(\Box A \vee \Box B)$
 $\left[\begin{array}{l} \Box A \\ \Box A \vee \Box B \\ \neg \Box A \\ \Box A \\ \Box A \vee \Box B \\ \Box A \\ \neg \Box B \\ \Box B \\ \Box B \text{ (}\vee E\text{)}$
 $\Box B \text{ (}\Box I\text{)}$
 $\Box A \vee \Box B$
 $\neg \neg(\Box A \vee \Box B)$
 $\Box A \vee \Box B$

\rightarrow
 $\left[\begin{array}{l} \Box B \\ \Box B \rightarrow \Box A \text{ (}\rightarrow I\text{)}$
 $\neg \Box B \text{ (}\ast\text{)} \vee \neg \Box B$
 $\neg \neg \Box B$
 $\Box B$
 $\Box A \rightarrow \Box B$
 $\neg \neg(\dots)$
 $\Box(\Box A \rightarrow \Box B) \vee \Box(\Box B \rightarrow \Box A)$