HOMEWORK I

To be handed in on Thursday, November 20. Collaboration is encouraged, but you may NOT copy another students solutions. It is important that you show all your work and write complete proofs. There are 4 problems for a total of 20 points. If you are stuck on a problem you may ask me for hints.

(1) Find all integer solutions to the Diophantine equation $53x + 31y = 2$.
(2) Solve the system of congruences

$$\begin{aligned} x &\equiv 1 \mod 2 \\ x &\equiv 2 \mod 3 \\ x &\equiv 3 \mod 5 \\ x &\equiv 4 \mod 7 \end{aligned}$$

(3) Solve the system of congruences

$$\begin{aligned} x &\equiv 13 \mod 21 \\ x &\equiv 7 \mod 15 \end{aligned}$$

Note that $\mathrm{GCD}(21, 15) \neq 1$.
(4) Let G be a finite group with exactly n elements. Prove that $2 \mid n$ if and only if there is an element in G of order 2. *Hint.* Use Lagrange's theorem and a counting argument.