SF2722 Differential geometry, spring 2009

Homework problems 2 To be handed in 14/4

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1 For $\alpha \in \mathbb{R}$ define open sets in $\mathbb{R}^3$ by

$$U_\alpha = \mathbb{R}^3 \setminus \{(t\cos\alpha, t\sin\alpha, s) : t \geq 0, s \in \mathbb{R}\},$$

$$V = (0, 2\pi) \times (-\pi/2, \pi/2) \times (0, \infty), \quad W = (0, 2\pi) \times (0, \infty) \times \mathbb{R},$$

and for $\alpha, \beta \in \mathbb{R}$ define coordinate charts on $\mathbb{R}^3$ as follows:

- Cartesian coordinates $(\mathbb{R}^3, f)$, where $f : \mathbb{R}^3 \to \mathbb{R}^3$ is the identity map.
- Spherical coordinates (U_α, g_α), where $g_\alpha : U_\alpha \to V$ is defined by

$$g_\alpha^{-1}(\varphi, \theta, \rho) = (\rho\cos(\varphi + \alpha)\cos\theta, \rho\sin(\varphi + \alpha)\cos\theta, \rho\sin\theta).$$

- Cylindrical coordinates (U_β, h_β), where $h_\beta : U_\beta \to W$ is defined by

$$h_\beta^{-1}(\psi, r, z) = (r\cos(\psi + \beta), r\sin(\psi + \beta), z).$$

Show that all these charts are contained in the same smooth structure on $\mathbb{R}^3$.

2 Find a vector field on the odd-dimensional sphere $S^{2k+1} \subset \mathbb{C}^{k+1}$ which is nowhere zero (*Hint:* complex multiplication $p \mapsto e^{it}p$ defines a flow on S^{2k+1}). Find a vector field on S^2 (or S^{2k}) with exactly one zero. Further, solve the same problem with S^n replaced by real projective space P^n.

3 The smooth Jordan curve theorem: Solve exercises 3.9.20, 3.9.22, 3.9.23 in the book.

4 Use the Brouwer fixed point theorem to prove that any 3×3-matrix with positive entries has an eigenvector with positive coordinates.