KTH Teknikvetenskap

SF2723 Topics in Mathematics - Matrix groups
Homework Assignment 1
2008-09-03

The solutions should be handed in no later than September 10, 2008. The final grade will be based upon the total score on the homework and on the oral exam. The total maximal score on the homework assignments is 200 and in order to pass, at least 100 is required.

In order to get full score on each problem, the written presentation of the solution should be clear and the arguments easy to follow.

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1. Which of the following maps $\mathrm{Gl}_n(\mathbf{C}) \longrightarrow \mathrm{Gl}_n(\mathbf{C})$ are group homomorphisms? In each case, give a proof or a counter-example.

   a) $X \mapsto {}^tX$.

   b) $X \mapsto \bar{X}$ (*complex conjugation*).

   c) $X \mapsto {}^tX^{-1}$.

   d) $X \mapsto X^{-1}$.

   e) $X \mapsto P^{-1}XP$, where P is in $\mathrm{Gl}_n(\mathbf{C})$.

   (5)

2. Does the set $\{X \in \mathrm{Gl}_2(\mathbf{R}) \mid X^2 = I\}$ form a subgroup of $\mathrm{Gl}_2(\mathbf{R})$? Give a proof or a counter-example. **(2)**

3. Show that for any orthogonal matrix A over the complex numbers, the map $X \mapsto {}^tAXA$ defines a group isomorphism

$$\mathrm{G}_S(\mathbf{C}) \longrightarrow \mathrm{G}_{{}^tASA}(\mathbf{C}).$$

   (4)

4. Show that $\mathrm{Gl}_1(\mathbf{C})$ is isomorphic to $\mathbf{C}^*$ and determine all finite subgroups of these groups. **(4)**