



KTH Teknikvetenskap

SF2723 Topics in Mathematics - Matrix groups
Homework Assignment 10
2008-11-12

The solutions should be handed in no later than November 19, 2008. The final grade will be based upon the total score on the homework and on the oral exam. The total maximal score on the homework assignments is 200 and in order to pass, at least 100 is required.

In order to get full score on each problem, the written presentation of the solution should be clear and the arguments easy to follow.

1. Show that the exponential map defined for the Lie group of matrices of the form

$$\begin{pmatrix} 1 & {}^t x & y \\ 0 & I_n & z \\ 0 & 0 & 1 \end{pmatrix}, \quad \text{where } x, z \in \mathbb{C}^n \text{ and } y \in \mathbb{C},$$

is polynomial and globally invertible. (5)

2. Determine the isomorphism classes of connected Lie subgroups of $\mathrm{Gl}_2(\mathbb{C})$ using the correspondence between the Lie algebra and the Lie group given by the exponential map. (*Hint:* Which subspaces of $\mathfrak{gl}_2(\mathbb{C}) \cong T_e \mathrm{Gl}_2(\mathbb{C})$ are Lie subalgebras of $\mathfrak{gl}_2(\mathbb{C})$?) (10)