SF2723 Topics in Mathematics - Matrix groups
Homework Assignment 12
2008-11-26

The solutions should be handed in no later than December 3, 2008. The final grade will be based upon the total score on the homework and on the oral exam. The total maximal score on the homework assignments is 200 and in order to pass, at least 100 is required.

In order to get full score on each problem, the written presentation of the solution should be clear and the arguments easy to follow.

We can cover projective space $\mathbb{P}^n_k$ by affine spaces $\mathbb{A}^n_k$ in the same way as we defined projective space as a manifold. The projective varieties in projective space will be those subsets whose restriction to these affine spaces are affine varieties.

1. Let $k = \mathbb{C}$ and consider the projective plane curves given by a homogeneous quadratic polynomial in three variables. Characterize these curves up to projective isomorphism. **(7)**

2. Let $k = \mathbb{R}$ and consider the family of elliptic plane curves given by the equation

$$y^2 z = x^3 - axz^2$$

for $a \in \mathbb{R}^*$. Give a picture of this curve in the three affine charts. **(4)**

3. Does $\mathbb{P}^1_\mathbb{C}$ have any structure as an algebraic group? **(4)**