KTH Teknikvetenskap

SF2723 Topics in Mathematics - Matrix groups
Homework Assignment 13
2008-12-03

The solutions should be handed in no later than December 10, 2008. The final grade will be based upon the total score on the homework and on the oral exam. The total maximal score on the homework assignments is 200 and in order to pass, at least 100 is required.

In order to get full score on each problem, the written presentation of the solution should be clear and the arguments easy to follow.

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1. Consider the twisted cubic in $\mathbb{P}^3_{\mathbb{C}}$ as the image of the map $\Phi : \mathbb{P}^1 \longrightarrow \mathbb{P}^3$ given by $\Phi(s : t) = (s^3 : s^2t : st^2 : t^3)$. Show that this gives an example where isomorphic projective varieties have non-isomorphic homogeneous coordinate rings. **(5)**

2. The Grassmannian of lines in projective space can be seen as the grassmannian of two-dimensional subspaces of $\mathbb{C}^4$. The Plücker embedding sends the 2×4-matrices giving representatives for elements in $\mathrm{Grass}(2, 4)$ into the six 2×2-minors. In this way, we get an embedding of $\mathrm{Grass}(2, 4)$ as a hypersurface in $\mathbb{P}^5_{\mathbb{C}}$. Determine the homogeneous equation defining it. **(5)**

3. The non-zero complex numbers, $\mathbb{C}^*$, form an algebraic group under multiplication.

   (a) Determine the coordinate rings $\mathbb{C}[\mathbb{C}^*]$ and $\mathbb{C}[\mathbb{C}^* \times \mathbb{C}^*]$.

   (b) Determine the $\mathbb{C}$-algebra homomorphisms

   $$\mathbb{C}[\mathbb{C}^*] \longrightarrow \mathbb{C}[\mathbb{C}^* \times \mathbb{C}^*]$$

   and

   $$\mathbb{C}[\mathbb{C}^*] \longrightarrow \mathbb{C}[\mathbb{C}^*]$$

   corresponding to multiplication and inverse, respectively.

   (5)