KTH Teknikvetenskap

SF2723 Topics in Mathematics - Matrix groups
Homework Assignment 2
2008-09-10

The solutions should be handed in no later than September 17, 2008. The final grade will be based upon the total score on the homework and on the oral exam. The total maximal score on the homework assignments is 200 and in order to pass, at least 100 is required.

In order to get full score on each problem, the written presentation of the solution should be clear and the arguments easy to follow.

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1. Let k be a field with three elements, i.e., $k \cong \mathbb{F}_3 \cong \mathbb{Z}/3\mathbb{Z}$. Determine the order of the matrix groups $\mathrm{Gl}_2(k)$, $\mathrm{Sl}_2(k)$, $\mathrm{O}_2(k)$, $\mathrm{SO}_2(k)$ and $\mathrm{Sp}_2(k)$. Are any of these groups isomorphic to eachother? **(6)**

   If S is a set and G is a group, we say that G *acts* on S if there is a group homomorphism $\Phi : G \longrightarrow \mathfrak{S}_S$. We often write $g.x$ for $\Phi(g)(x)$. For $x \in S$ we have an *orbit*,
   $$Gx = \{y \in S \mid \exists g \in G \colon y = g.x\},$$
   and a *stabilizer*,
   $$G_x = \{g \in G \mid g.x = x\}.$$
   If G is finite, we have that
   $$|G| = |Gx| \cdot |G_x|.$$
   The matrix groups over k naturally acts on the set k^n by matrix multiplication, when k^n is seen as the set of column matrices of size $n \times 1$. This action can be used to compute the order of the classical matrix groups over finite fields, k.

2. Compute the order of $\mathrm{Gl}_n(k)$, where k is a finite field of odd order q. (Hint: *Look at the orbit and stabilizer of the first standard basis vector* ${}^t(1, 0, \ldots, 0)$ *and make an induction over* n.) **(5)**

3. Write any element of $\mathrm{Sl}_2(\mathbb{R})$ as a product of elementary matrices. **(3)**