KTH Teknikvetenskap

SF2723 Topics in Mathematics - Matrix groups
Homework Assignment 4
2008-09-24

The solutions should be handed in no later than October 1, 2008. The final grade will be based upon the total score on the homework and on the oral exam. The total maximal score on the homework assignments is 200 and in order to pass, at least 100 is required.

In order to get full score on each problem, the written presentation of the solution should be clear and the arguments easy to follow.

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1. Compute
$$\exp\begin{pmatrix} 1 & 1 \\ -1 & 3 \end{pmatrix}.$$
(3)

2. Let V be a finite-dimensional vector space over $k = \mathbb{R}$ or $k = \mathbb{C}$ and let $\langle\,,\rangle$ be a non-degenerate bilinear form on V.
   a) Show that the exponential map is well-defined from $\mathrm{Hom}_k(V, V)$ to $\mathrm{Gl}(V)$.
   b) Suppose that $\langle\,,\rangle$ is symmetric and let x in V be a vector with $\langle x, x\rangle \neq 0$. Determine $\exp(s_x)$.
   c) Suppose that $\langle\,,\rangle$ is alternating. Determine $\exp(\Phi)$ where Φ is the transvection given by $\Phi(y) = y + \lambda\langle y, x\rangle x$.

(2,3,3)

3. Use the fact that diagonalizable matrices are dense in $\mathrm{M}_n(\mathbb{C})$ to prove the *Cayley–Hamilton Theorem* for real and complex matrices. **(3)**