KTH Teknikvetenskap

SF2723 Topics in Mathematics - Matrix groups
Homework Assignment 6
2008-10-08

The solutions should be handed in no later than October 15, 2008. The final grade will be based upon the total score on the homework and on the oral exam. The total maximal score on the homework assignments is 200 and in order to pass, at least 100 is required.

In order to get full score on each problem, the written presentation of the solution should be clear and the arguments easy to follow.

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1. Let Z be the analytic set in $\mathbb{C}^2$ given by the zeroes of the polynomial $y^2 + x^2(x+1)$.
   (a) Determine the locus where the normal space of Z has dimension 1 and the locus where it has dimension is 0.
   (b) For each point in Z where the normal space has dimension 1, give an explicit open set U in $\mathbb{C}^2$ and an explicit analytic bijection $\Phi : V \longrightarrow U$, where V is open in $\mathbb{C}^2$, satisfying the conditions of Theorem 3-2.3.

(7)

2. Let $X = \mathbb{Z}$ and for $a \in \mathbb{Z} \setminus \{0\}$ and $b \in \mathbb{Z}$ define $X_{a,b} = \{ax + b \mid x \in \mathbb{Z}\}$. Let $\mathcal{U}$ consist of all unions of sets of the form $X_{a,b}$.
   (a) Show that $\mathcal{U}$ is a topology on X.
   (b) Show that all the sets $X_{a,b}$, for $a, b \in \mathbb{Z}$, are both open and closed.
   (c) Let $P \subseteq \mathbb{Z}$ be the set of prime numbers. Show that $\bigcup_{p \in P} X_{p,0} = X \setminus \{1, -1\}$.
   (d) Show that $\{-1, 1\}$ is not open, and that this implies that P is infinite.

(7)