Homework assignment 1
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Hand calculation of the expansion in the Haaar system $\{h_{kj}\}$,

The Haar System of function is given by

$$h_{kj} = 2^{j/2} h(2^j x - k),$$

where

$$h(x) = \begin{cases} -1 & \text{for } 0 \le x < \frac{1}{2}. \\ 1 & \text{for } \frac{1}{2} < x \le 1. \end{cases}$$

Given is the following function f on the interval $I_0 = [0, 1]$.

$$f(x) = \begin{cases} x & \text{for } 0 \le x \le \frac{9}{16}, \\ \frac{1}{4} & \text{for } \frac{9}{16} < x \le 1. \end{cases}$$

In the following $N = 4$. The drawing of the graphs below could be done in the same diagram if using different colours.

1. Draw the graph of f.

2. Divide the interval I_0 into 2^N equally size intervals. Do the projection f into a step-wise constant auction f_0 on those intervals.(Minimising the norm $||f - f_0||$.). Draw the graph of f_0.

3. Make a Haar wavelet expansion of f_0 using functions h_{kj}, $j = N-1, \ldots, 0$ were and, the characteristic function χ_{I_0} of the unit interval.
   List the corresponding coefficients.
   (Don't expand $\sqrt{2}$ numerically)

4. Verify that f_0 can be exactly reconstructed from those wavelet coefficients.

5. Replace the smallest 8 (of totally 16) coefficients by zero and make an approximate reconstruction f_A from the remaining 8 coefficients.
   Draw the graph of f_A.

6. Error estimation: Estimate the the L^2 norms $||f||, ||f - f_0||$, $||f_0 - f_A||$ and $||f - f_A||$.

7. Use a hand calculator to compute the signal-to-noise ratio

$$SNR = -20 \quad {}^{10} \log \frac{||f - f_A||}{||f||}$$