# Homework nr 5

Handed out November 25, 2009
To be handed in December 9, 2009

**The local cosine transform**

The data consists of $N$ sample points. We assume the sampling is done at half points between integers: $n+\frac{1}{2}$. The sampling interval is broken up into intervals $[a_{j-1}, a_j]$ where $a_j$ are integers.
Around each interval endpoint $a_j$, $0 < j < 2^k$ there is a transition interval $[a_j - b_j, a_j+b_j]$ where windows overlap. Here $b_j$ is an integer and $b_j+b_{j-1} \leq a_j-a_{j-1}$ so that the transition intervals do not overlap. (For simplicity we may assume that $N = 2^m$ and there are $2^k$ intervals of lengths $L_j = a_j - a_{j-1} = L = 2^{(m-k)}$ ,where $0 < k < m$ are fixed and also that all $b_j$ are equal: $b_j = b$)

1. Build a folding matrix (and its inverse: the unfolding matrix).
   This will be a matrix $\{c_{jk}\}$ of size $2b\times 2b$ with non-zero entries only on two crossing diagonals. There values are given by the $2 \times 2$ rotation matrices

   $$\begin{pmatrix} c_{j,j} & c_{j,2b-j+1} \\ c_{2b-j+1,j} & c_{2b-j+1,2b+j-1} \end{pmatrix} = \begin{pmatrix} \cos\alpha_j & -\sin\alpha_j \\ \sin\alpha_j & \cos\alpha_j \end{pmatrix} = \begin{pmatrix} \cos\alpha_j & -\cos\alpha_{2b+1-j} \\ \cos\alpha_{2b+1-j} & \cos\alpha_j \end{pmatrix}$$

   for $j = 1, \ldots, b$ where the rotation angles

   $$\alpha_j = \frac{\pi}{2} P\left(\frac{j-b-\frac{1}{2}}{b}\right) \text{ for } j = 1, \ldots, 2b.$$

   We may chose $P(t) = (t^3 - 3t + 2)/4$.

2. Folding the data in all the transition intervals. The matrix above applies to the data in the transition interval for each transition interval. Use this matrix to modify those data points which belongs to these transition intervals.

3. After this may organise the folded data (using reshape) into with $2^k$ columns$f_j()$ each of length $2^{m-k}$.

   Since sampling points are at half integer points and also we looking at half integer frequencies the cosine transform will be

   $$\begin{aligned} \hat{f}_j(k) &= \tfrac{2}{\sqrt{L}} \textstyle\sum_{n=1}^{L} f_j(n) \cos\{\pi(k-\tfrac{1}{2})(n-\tfrac{1}{2})/L\} \\ &= \text{Real part } \left[\tfrac{2}{\sqrt{L}} \textstyle\sum_{n=1}^{L} f_j(n) \exp\{i\pi(k-\tfrac{1}{2})(n-\tfrac{1}{2})/L\}\right] \end{aligned}$$

4. Compare the formula above with Matlabs fft by using 'help fft'. Find factors $A_n$ and $B_k$ so that the formula's will be consistent with $x(n) = A_n f(n)$ and $\hat{f}(k) = B_k X(k)$ , where $x(n)$ and $X(k)$ are given by the formula 'help fft' in Matlab.

5. Try to Inverse the process by first taken the inverse of the cosine transform -by taking inverse fft with matlab involving the weight factors $A_n$ and $B_k$ above .
   Finally use the unfolding matrix to modify the data at the transition intervals.

6. Try the transform on the data file guitar.wav at my website http://www.kth.se/ jostromb/SF2702 .
   You may choose a subsequence of suitable length. Suggestion set $b = 8$ and $L = 64$.