MATEMATISKA INSTITUTIONEN
KTH
Kursledare: Svante Linusson
telefon 08-790 94 44

Hand in problems in SF2704 Graph Theory HT2009, set 4

Be sure to write solutions with clear arguments that are easy to follow. You should try to have a level of details so your solution would be understandable to other students. Staple you solution together in the top left corner and write down your solutions in order. Write your name in the top right corner.

Hederskodex (Code of conduct): It is assumed that:
-you shall solve the problems on your own and write down your own solution
-if you in spite of this are using something you have gotten from somewhere else for some reason (a friend, a book or the internet etc.) you should give a reference to the source.

Your solutions to the problems are due January 12th at 10.00.
(Come to my office, put them in the post box at the entrance or send them by email.)
PLEASE: Motivate your solutions clearly!

1. Prove that the Ramsey number $R(4,5) \leq 31$. You may use any fact from the handout on Ramsey theory (except the fact that $R(4,5) = 25$).

2. What is the expected number of K^r-subgraphs in $\mathcal{G}(n,p)$?
(This is exercise 11.3 in Diestel.)

3. Let $G = (V, E)$ be a graph and let $m := ||G||$ be the number of edges. Show that G contains a k-partite subgraph with $m(k-1)/k$ edges.
(Hint: Consider a random coloring of $V(G)$ with k colors and study the expected number of edges with the same color in both endvertices.)

4. Let $\epsilon > 0$ and $p = p(n) > 0$, and let $r(n) \geq (1+\epsilon)(2\ln n)/p$ be an integer valued function. Show that almost no graph in $\mathcal{G}(n,p)$ contains $r(n)$ independent vertices.
(This is exercise 11.10 in Diestel.)

5. (This exercise should be more difficult) Define a *random interval graph* G_n with vertex set $\{1, \ldots, n\}$ by choosing a random* partition $\{a_1, b_1\}, \{a_2, b_2\}, \ldots, \{a_n, b_n\}$ of $\{1, 2, \ldots, 2n\}$, say, with $a_i < b_i$ and join vertices i and j in G_n if $[a_i, b_i] \cap [a_j, b_j] \neq \emptyset$.

   (a) Compute the expected number of edges in G_n.

   (b) Show that almost every G_n is connected

   * Every possible such partition should be equally likely.

Lycka till!

Svante