

**SF2735 Homologisk Algebra**  
**Exercise set 2**

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The solutions to these exercises are to be handed in no later than Thursday, 22th of October. Please pay attention to the presentation as well as the arguments given in the solutions.

**Exercise 1**

Prove/disprove the following four statements. The  $\mathbf{Z}$ -module  $\mathbf{Q}$  is a) finitely generated, b) free, c) projective, d) injective.

**Exercise 2**

Let  $M$  and  $N$  be two fixed  $A$ -modules. Show that the set of  $A$ -modules  $E$  that fits in a short exact sequence

$$0 \rightarrow N \rightarrow E \rightarrow M \rightarrow 0$$

has an  $A$ -module structure, and as such it is  $\text{Ext}_A^1(M, N)$ .

**Exercise 3**

We have the short exact sequence

$$0 \rightarrow \mathbf{Z}(4) \rightarrow \mathbf{Z}/(16) \rightarrow \mathbf{Z}/(4) \rightarrow 0,$$

of  $\mathbf{Z}$ -modules. By the previous exercise the sequence corresponds to an element in the group  $\text{Ext}_{\mathbf{Z}}^1(\mathbf{Z}/(4), \mathbf{Z}/(4))$ . Which short exact sequence corresponds to its inverse?