1. $f(x) = xe^{-x^2/2}$.

$f'(x) = e^{-x^2/2} + x\cdot(-x)e^{-x^2/2} = (1-x^2)e^{-x^2/2}$.

$f'(x) = 0 \Leftrightarrow x = \pm 1$

$f(-1) = -e^{-1/2} = -\frac{1}{\sqrt{e}}$ $\qquad f(1) = e^{-1/2} = \frac{1}{\sqrt{e}}$

$\lim_{x\to\pm\infty} f(x) = \lim_{x\to\pm\infty} \frac{x}{e^{x^2/2}} \to 0$

x		-1		1	
$f'(x)$	$-$	0	$+$	0	$-$
$f(x)$	$\searrow$		$\nearrow$		$\searrow$

SVAR: STÖRSTA VÄRDE ÄR $\frac{1}{\sqrt{e}}$, MINSTA VÄRDE ÄR $-\frac{1}{\sqrt{e}}$

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2. $\displaystyle\int_1^e \frac{(\ln x)^4}{x}\,dx = I$

a) $\left.\begin{array}{l} u = \ln x \\ du = \frac{1}{x}dx \\ x=1 \Rightarrow u=0 \\ x=e \Rightarrow u=1 \end{array}\right\} \Rightarrow I = \displaystyle\int_0^1 u^4\,du$ $\qquad$ SVAR: $\displaystyle\int_0^1 u^4\,du$

b) $\displaystyle\int_0^1 u^4\,du = \left[\frac{u^5}{5}\right]_0^1 = \frac{1}{5}$ $\qquad$ SVAR: $\frac{1}{5}$

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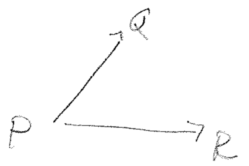
3. $P = (1,0,1)$ $\qquad Q = (2,1,3)$ $\qquad R = (0,-1,0)$

a)



$\overrightarrow{PQ} = (2,1,3) - (1,0,1) = (1,1,2)$

$\overrightarrow{PR} = (0,-1,0) - (1,0,1) = (-1,-1,-1)$

$\bar{n}_\pi = \begin{vmatrix} \bar{e}_x & \bar{e}_y & \bar{e}_z \\ 1 & 1 & 2 \\ -1 & -1 & -1 \end{vmatrix} = (-1+2,\ -2+1,\ -1+1) = (1,-1,0)$

EKV. FÖR PLANET π: $(1,-1,0)(x-1, y, z-1) = 0$

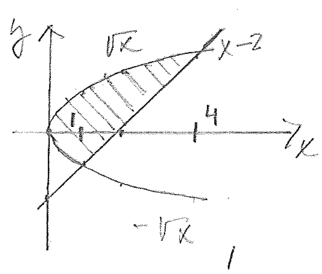
$x-1-y = 0 \Leftrightarrow x-y-1 = 0$

b) $A_{PQR} = |\overrightarrow{PQ} \times \overrightarrow{PR}|/2 = \sqrt{1^2+(-1)^2}/2 = \sqrt{2}/2$.

c) $\ell: \begin{cases} x = 0 + t \\ y = -1 - t \\ z = 0 \end{cases}$

SVAR: a) $x-y=1$
b) $\sqrt{2}/2$
c) $x=t,\ y=-1-t,\ z=0$

4.



SKÄRNINGSPUNKTER:

$$\begin{cases} y=\sqrt{x} \\ y=-\sqrt{x} \end{cases} \Rightarrow x=0 \qquad \begin{cases} y=\sqrt{x} \\ y=x-2 \end{cases} \Rightarrow \sqrt{x}=x-2$$

$$x = x^2-4x+4$$
$$x^2-5x+4=0$$
$$x=\frac{5}{2}\pm\sqrt{\frac{25}{4}-\frac{16}{4}}$$
$$x=1,\ x=4$$

$$\text{AREAN} = \int_0^1 (\sqrt{x}-(-\sqrt{x}))\,dx + \int_1^4 (\sqrt{x}-(x-2))\,dx =$$

$$= \int_0^1 2\sqrt{x}\,dx + \int_1^4 (\sqrt{x}-x+2)\,dx = \left[2\cdot\frac{2}{3}x\sqrt{x}\right]_0^1 + \left[\frac{2}{3}x\sqrt{x}-\frac{x^2}{2}+2x\right]_1^4$$

$$= \frac{4}{3}+\frac{2}{3}\cdot 4\cdot 2-8+8-\frac{2}{3}+\frac{1}{2}-2 = \frac{18}{3}-\frac{3}{2} = 6-\frac{3}{2} = \frac{9}{2}$$

SVAR: $\frac{9}{2}$ a.e

5.

$$y''+4y'+5y = 8\cos x \quad (*)$$

HOMOGEN LÖSN: $y''+4y'+5y=0$

KAR. EKV. $r^2+4r+5=0 \Leftrightarrow r=-2\pm\sqrt{4-5}$

$$r=-2\pm i$$

$$y_h = e^{-2x}(A\cos x + B\sin x)$$

PART. LÖSN: SÄTT $y_p = a\cos x + b\sin x$

$$y_p' = -a\sin x + b\cos x \qquad y_p'' = -a\cos x - b\sin x$$

INSATT I (*): $-a\cos x - b\sin x - 4a\sin x + 4b\cos x$

$$+5a\cos x + 5b\sin x \equiv 8\cos x$$

$$(4a+4b)\cos x + (4b-4a)\sin x \equiv 8\cos x$$

$$4b-4a=0 \Leftrightarrow a=b$$

$$4a+4b=8 \Rightarrow 8a=8 \Leftrightarrow a=1=b$$

$$y_p = \cos x + \sin x.$$

$$y = y_p + y_h = \cos x + \sin x + e^{-2x}(A\cos x + B\sin x)$$

SVAR: $y = \cos x + \sin x + e^{-2x}(A\cos x + B\sin x)$

6. a) $f(x) = \ln(1+x) = x - \dfrac{x^2}{2}$

b) $\ln(1,1) = \ln(1+0,1) \approx 0,1 - \dfrac{(0,1)^2}{2} = 0,1 - \dfrac{0,01}{2} = 0,1 - 0,005$
$= 0,095.$

c) $R_2 = \left|\dfrac{x^3}{3}\right| = \dfrac{(0,1)^3}{3} = \dfrac{0,001}{3} < 0,001$

SVAR: a) $\ln(1+x) = x - \dfrac{x^2}{2}$ b) $0,095$ c) JA

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7. $y = \dfrac{e^x + e^{-x}}{2} \qquad 0 \le x \le 1 \qquad L = \displaystyle\int_0^1 \sqrt{1+(f'(x))^2}\, dx$

$f'(x) = \dfrac{e^x - e^{-x}}{2} \qquad 1 + (f'(x))^2 = 1 + \left(\dfrac{e^x - e^{-x}}{2}\right)^2 =$

$= 1 + \dfrac{e^{2x} - 2 + e^{-2x}}{4} = \dfrac{4 + e^{2x} - 2 + e^{-2x}}{4} = \dfrac{2 + e^{2x} + e^{-2x}}{2} =$

$= \left(\dfrac{e^x + e^{-x}}{2}\right)^2 \qquad \sqrt{1+(f'(x))^2} = \sqrt{\left(\dfrac{e^x + e^{-x}}{2}\right)^2} = \dfrac{e^x + e^{-x}}{2}$

$L = \displaystyle\int_0^1 \dfrac{e^x + e^{-x}}{2}\, dx = \left[\tfrac{1}{2}\left(e^x - e^{-x}\right)\right]_0^1 = \tfrac{1}{2}(e - e^{-1} - 1 + 1) =$

$= \tfrac{1}{2}\left(e - \tfrac{1}{e}\right)$ SVAR: $\tfrac{1}{2}\left(e - \tfrac{1}{e}\right)$ l.e

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8. $y = (x^2 - 2)e^{-2x}$

NOLLSTÄLLEN: $(x^2-2)e^{-2x} = 0 \iff x^2 - 2 = 0 \iff x = \pm\sqrt{2}$

$y' = 2x e^{-2x} - 2(x^2-2)e^{-2x} = e^{-2x}(2x - 2x^2 + 4) = 2e^{-2x}(x - x^2 + 2)$

$y' = 0 \Rightarrow x - x^2 + 2 = 0 \qquad x^2 - x - 2 = 0 \qquad x = \tfrac{1}{2} \pm \sqrt{\tfrac{1}{4} + \tfrac{8}{4}}$

$x = \tfrac{1}{2} \pm \tfrac{3}{2} \qquad x = 2, \quad x = -1$

x		-1		2	
$f'(x)$	$-$	0	$+$	0	$-$
$f(x)$	$\searrow$	LOK MIN	$\nearrow$	LOK MAX	$\searrow$

LOK. MIN i $f(-1) = -e^2$

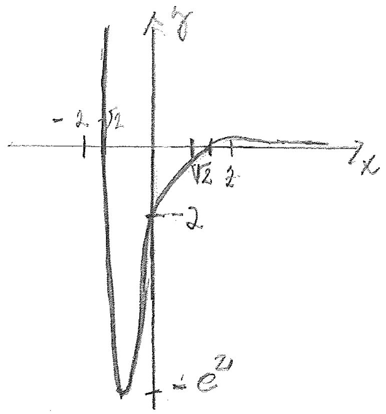
LOK. MAX i $f(2) = 2e^{-4}$

FORTS.

8. FORTS.

ASYMPTOTER: $\lim_{x\to\infty} (x^2-2)e^{-2x} = \lim_{x\to\infty} \frac{x^2-2}{e^{2x}} \to 0$

$\therefore$ $y=0$ ÄR ASYMPTOT.



SVAR: NOLLSTÄLLEN DÅ $x=\pm\sqrt{2}$

LOK. MAX I $(2, 2e^{-4})$

LOK. MIN I $(-1, -e^2)$

$y=0$ ÄR ASYMPTOT

9 a) C = CIRKELNS EKV: $x^2+y^2=R^2$ P: (x_0, y_0)

IMPLICIT DER. MED AVSEENDE PÅ x: $2x+2y\cdot y'=0 \Leftrightarrow y'=-\frac{x}{y}$

$y'(x_0,y_0) = \frac{-x_0}{y_0}$

TANG. I P: $y-y_0 = \frac{-x_0}{y_0}(x-x_0)$;

$y\cdot y_0 - y_0\cdot y_0 = -x\cdot x_0 + x_0\cdot x_0$

$x\cdot x_0 + y\cdot y_0 = (x_0)^2+(y_0)^2 = R^2 \quad \therefore\ x x_0 + y\cdot y_0 = R^2$

V.S.V.

b) E = ELLIPS: $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ P: (x_0, y_0) LIGGER PÅ E DVS $\frac{x_0^2}{a^2}+\frac{y_0^2}{b^2}=1$

IMPLICIT DERV MED AVSEENDE PÅ x: $\frac{2x}{a^2}+\frac{2y'y}{b^2}=0 \Rightarrow$

$y' = -\frac{x}{a^2}\cdot\frac{b^2}{y} = -\frac{x}{y}\cdot\frac{b^2}{a^2} \qquad y'(x_0,y_0) = -\frac{x_0}{y_0}\cdot\frac{b^2}{a^2}$

TANG. I P: $y-y_0 = -\frac{x_0}{y_0}\cdot\frac{b^2}{a^2}(x-x_0)$;

$a^2(y\cdot y_0 - y_0^2) = b^2(-x x_0 + x_0^2)$; $\quad b^2 x x_0 + a^2 y\cdot y_0 = b^2 x_0^2 + a^2 y_0^2$;

$\left(\frac{x_0^2}{a^2}+\frac{y_0^2}{b^2}=1\right.$, DIVIDERA MED $\left.a^2b^2\right)$ $\quad \frac{x x_0}{a^2}+\frac{y y_0}{b^2} = \frac{x_0^2}{a^2}+\frac{y_0^2}{b^2} = 1$ V.S.V.

10. a) $f(x) = \frac{\sin x}{x} \quad x \neq 0$

$f(0) = c$

$\lim_{x \to 0} \frac{\sin x}{x} = 1$ STANDARDGRÄNSVÄRDE

$\therefore$ $f(0)=1$ GÖR $f(x)$ KONTINUERLIG. <u>SVAR</u>: $c=1$

b) $f'(x) = \frac{x\cos x - \sin x}{x^2}$ DÅ $x \neq 0$.

VI MÅSTE ANVÄNDA DERIVATANS DEFINITION FÖR ATT BESTÄMMA $f'(0)$

$$f'(0) = \lim_{h \to 0} \frac{f(0+h) - f(0)}{h} = \lim_{h \to 0} \frac{\frac{\sin h}{h} - 1}{h} = \lim_{h \to 0} \frac{\sin h - h}{h^2}$$

$$= \left| \text{"}\frac{0}{0}\text{" l'H} \right| = \lim_{h \to 0} \frac{\cos h - 1}{2h} = \left| \text{"}\frac{0}{0}\text{" l'H} \right| = \lim_{h \to 0} \frac{-\sin h}{2} = 0$$

$\therefore$ $f'(0) = 0$

$$f''(0) = \lim_{h \to 0} \frac{f'(0+h) - f'(0)}{h} =$$

$$= \lim_{h \to 0} \frac{\frac{h\cos h - \sin h}{h^2} - 0}{h} =$$

$$= \lim_{h \to 0} \frac{h\cos h - \sin h}{h^3} = \left| \begin{matrix} \text{MacLaurin} \\ \text{utv} \end{matrix} \right| =$$

$$= \lim_{h \to 0} \frac{h(1 - \frac{h^2}{2!} + O(h^4)) - (h - \frac{h^3}{3!} + O(h^5))}{h^3} =$$

$$= \lim_{h \to 0} \frac{h - \frac{h^3}{2} - h + \frac{h^3}{6} + O(h^4)}{h^3} = \lim_{h \to 0} \frac{h^3(\frac{1}{6} - \frac{1}{2})}{h^3} =$$

$$= \frac{1}{6} - \frac{1}{2} = \frac{1-3}{6} = -\frac{1}{3}$$

<u>SVAR</u>: $f'(0) = 0$, $f''(0) = -\frac{1}{3}$