

USEFUL FORMULAS

Trigonometric Identities

$$\cos(a + b) = \cos a \cos b - \sin a \sin b$$

$$\sin(a + b) = \sin a \cos b + \cos a \sin b$$

$$\cos\left(a - \frac{\pi}{2}\right) = \sin a$$

$$\cos^2 a = \frac{1 + \cos 2a}{2}$$

$$\cos 2a = \cos^2 a - \sin^2 a$$

$$\cos a + \sin a = \sqrt{2} \cos\left(a - \frac{\pi}{4}\right)$$

$$\alpha \cos a + \beta \sin a = \sqrt{\alpha^2 + \beta^2} \cos(a - b), \text{ where } \cos b = \frac{\alpha}{\sqrt{\alpha^2 + \beta^2}} \text{ and } \sin b = \frac{\beta}{\sqrt{\alpha^2 + \beta^2}}$$

$$\cos(a - b) = \cos a \cos b + \sin a \sin b$$

$$\sin(a - b) = \sin a \cos b - \cos a \sin b$$

$$\sin\left(a + \frac{\pi}{2}\right) = \cos a$$

$$\sin^2 a = \frac{1 - \cos 2a}{2}$$

$$\sin 2a = 2 \sin a \cos a$$

$$\cos a + \sin a = \sqrt{2} \sin\left(a + \frac{\pi}{4}\right)$$

Complex Numbers

$$z = x + iy$$

$$\bar{z} = x - iy$$

$$\operatorname{Re}(z) = x, \operatorname{Im}(z) = y$$

$$z + \bar{z} = 2 \operatorname{Re}(z) = 2x \quad z - \bar{z} = 2i \operatorname{Im}(z) = 2iy \quad |z| = \sqrt{z \cdot \bar{z}} = \sqrt{x^2 + y^2}$$

$$|z|^2 = z \cdot \bar{z} = x^2 + y^2 \quad z^2 = x^2 - y^2 + 2ixy \quad \frac{1}{z} = \frac{\bar{z}}{z \cdot \bar{z}} = \frac{x - iy}{x^2 + y^2} \quad (z \neq 0)$$

Triangle Inequality: If z and w are any complex numbers, then

$$|z \pm w| \leq |z| + |w| \quad \text{and} \quad ||z| - |w|| \leq |z \pm w|$$

Euler's Identity and Related Identities: If x is any real number, then

$$e^{ix} = \cos x + i \sin x \quad e^{-ix} = \cos x - i \sin x \quad e^{-ix} = \overline{e^{ix}} \quad |e^{ix}| = 1$$

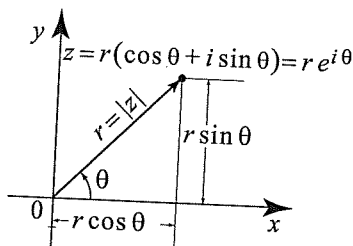
$$\cos x = \frac{e^{ix} + e^{-ix}}{2} \quad \sin x = \frac{e^{ix} - e^{-ix}}{2i} \quad (\cos nx + i \sin nx)^n = e^{inx} = \cos nx + i \sin nx$$

Polar Representation of Complex Numbers

$$z = x + iy = r e^{i\theta} = r(\cos \theta + i \sin \theta)$$

$$r = |z| = \sqrt{x^2 + y^2}$$

$$x = r \cos \theta, \quad y = r \sin \theta$$



Taylor Series Expansions: Let z be a real or complex number. Then

$$e^z = \sum_{n=0}^{\infty} \frac{z^n}{n!} \quad (\text{any } z) \quad \cos z = \sum_{n=0}^{\infty} (-1)^n \frac{z^{2n}}{(2n)!} \quad (\text{any } z) \quad \sin z = \sum_{n=0}^{\infty} (-1)^n \frac{z^{2n+1}}{(2n+1)!} \quad (\text{any } z)$$

$$\frac{1}{1-z} = \sum_{n=0}^{\infty} z^n \quad (|z| < 1) \quad \cosh z = \sum_{n=0}^{\infty} \frac{z^{2n}}{(2n)!} \quad (\text{any } z) \quad \sinh z = \sum_{n=0}^{\infty} \frac{z^{2n+1}}{(2n+1)!} \quad (\text{any } z)$$

Useful Integrals (Take $a \neq 0$ and b to be real, and m and n to be integers.)

Integrals Involving Trigonometric Functions

$$\int \cos(ax + b) dx = \frac{1}{a} \sin(ax + b) + C \qquad \int \sin(ax + b) dx = -\frac{1}{a} \cos(ax + b) + C$$

$$\int \cos^2(ax + b) dx = \frac{x}{2} + \frac{1}{4a} \sin(2(ax + b)) + C$$

$$\int \sin^2(ax + b) dx = \frac{x}{2} - \frac{1}{4a} \sin(2(ax + b)) + C$$

$$\int x \cos ax dx = \frac{1}{a^2} \cos ax + \frac{x}{a} \sin ax + C \qquad \int x \sin ax dx = \frac{1}{a^2} \sin ax - \frac{x}{a} \cos ax + C$$

$$\int x^2 \cos ax dx = \frac{2x \cos ax}{a^2} + \frac{a^2 x^2 - 2}{a^3} \sin ax + C$$

$$\int x^2 \sin ax dx = \frac{2x \sin ax}{a^2} - \frac{a^2 x^2 - 2}{a^3} \cos ax + C$$

$$\int x^m \cos ax dx = \frac{x^m \sin ax}{a} - \frac{m}{a} \int x^{m-1} \sin ax dx$$

$$\int x^m \sin ax dx = -\frac{x^m \cos ax}{a} + \frac{m}{a} \int x^{m-1} \cos ax dx$$

$$\int \cos ax \cos bx dx = \frac{\sin[(a-b)x]}{2(a-b)} + \frac{\sin[(a+b)x]}{2(a+b)} + C \quad (a^2 \neq b^2)$$

$$\int \sin ax \sin bx dx = \frac{\sin[(a-b)x]}{2(a-b)} - \frac{\sin[(a+b)x]}{2(a+b)} + C \quad (a^2 \neq b^2)$$

$$\int \cos ax \sin bx dx = \frac{\cos[(a-b)x]}{2(a-b)} - \frac{\cos[(a+b)x]}{2(a+b)} + C \quad (a^2 \neq b^2)$$

Integrals Involving Exponential Functions

$$\int x e^{ax+b} dx = \frac{e^{ax+b}}{a^2} (ax - 1) + C \qquad \int x^m e^{ax+b} dx = \frac{x^m e^{ax+b}}{a} - \frac{m}{a} \int x^{m-1} e^{ax+b} dx$$

$$\int e^{ax} \cos bx dx = \frac{e^{ax}}{a^2 + b^2} (a \cos bx + b \sin bx) + C \quad (a^2 + b^2 \neq 0)$$

$$\int e^{ax} \sin bx dx = \frac{e^{ax}}{a^2 + b^2} (a \sin bx - b \cos bx) + C \quad (a^2 + b^2 \neq 0)$$

Identities Involving Bessel Functions (Take $p \geq 0$, $a \neq 0$, $n = 0, 1, \dots$)

$$\frac{d}{dx} [J_0(x)] = -J_1(x)$$

$$\frac{d}{dx} [x^p J_p(x)] = x^p J_{p-1}(x)$$

$$\frac{d}{dx} [x^{-p} J_p(x)] = -x^{-p} J_{p+1}(x)$$

$$x J_p'(x) + p J_p(x) = x J_{p-1}(x)$$

$$x J_p'(x) - p J_p(x) = -x J_{p+1}(x)$$

$$J_{p-1}(x) - J_{p+1}(x) = 2 J_p'(x)$$

$$J_{p-1}(x) + J_{p+1}(x) = \frac{2p}{x} J_p(x)$$

$$J_n(x) = \frac{1}{\pi} \int_0^\pi \cos(n\theta - x \sin \theta) d\theta$$

$$\int x^{p+1} J_p(x) dx = x^{p+1} J_{p+1}(x) + C$$

$$\int x^{-p+1} J_p(x) dx = -x^{-p+1} J_{p-1}(x) + C$$

$$\int J_1(x) dx = -J_0(x) + C$$

$$\int x J_0(x) dx = x J_1(x) + C$$

$$\int J_{p+1}(x) dx = \int J_{p-1}(x) dx - 2 J_p(x)$$

$$x J_{p+1}(x) + p \int J_{p+1}(x) dx = \int x J_p(x) dx$$

$$\int J_{2n+1}(x) dx = -J_0(x) - 2 \sum_{k=1}^n J_{2k}(x) + C$$

$$\int_0^a x^{p+1} J_p\left(\frac{\alpha}{a} x\right) dx = \frac{a^{p+2}}{\alpha} J_{p+1}(\alpha)$$

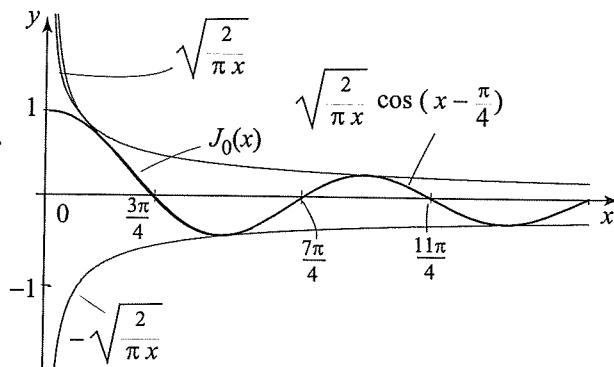
$$\int x J_{2n}(x) dx = x J_{2n+1}(x) - 2n J_0(x) - 4n \sum_{k=1}^n J_{2k}(x) + C$$

Zeros and Asymptotics of Bessel Functions (Take $n = 0, 1, 2, \dots$)

$$J_n(x) \sim \sqrt{\frac{2}{\pi x}} \cos\left(x - \frac{\pi}{4} - \frac{n\pi}{2}\right) + \mathcal{O}\left(\frac{1}{x^{3/2}}\right)$$

If α_k = k th positive zero of $J_n(x)$, then for large k

$$\alpha_k \approx \frac{\pi}{4} + \frac{(n+1)\pi}{2} + k\pi$$



Improper Integrals (Take $a \neq 0$.)

$$\int_{-\infty}^{\infty} \frac{\sin ax}{x} dx = \frac{\pi}{2}, \text{ if } a > 0; -\frac{\pi}{2} \text{ if } a < 0.$$

$$\int_0^{\infty} e^{-a^2 x^2} dx = \frac{\sqrt{\pi}}{2a}$$

$$\int_0^{\infty} e^{-a^2 x^2} \cos bx dx = \frac{\sqrt{\pi}}{2|a|} e^{-b^2/(4a^2)} \quad (b \neq 0)$$

$$\int_0^{\infty} \frac{\cos wx}{a^2 + x^2} dx = \frac{\pi}{2|a|} e^{-|wa|}$$

$$\int_{-\infty}^{\infty} \cos(x^2) dx = \int_{-\infty}^{\infty} \sin(x^2) dx = \sqrt{\frac{\pi}{2}}$$

Table of Fourier Transforms

$f(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} \hat{f}(\omega) e^{ix\omega} d\omega$		$\hat{f}(\omega) = \mathcal{F}(f)(\omega) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} f(x) e^{-i\omega x} dx$	
1.	$\begin{cases} 1 & \text{if } x < a \\ 0 & \text{if } x > a \end{cases}$		$\sqrt{\frac{2}{\pi}} \frac{\sin a\omega}{\omega}$
2.	$\begin{cases} 1 & \text{if } a < x < b \\ 0 & \text{otherwise} \end{cases}$		$\frac{i(e^{-ib\omega} - e^{ia\omega})}{\sqrt{2\pi}\omega}$
3.	$\begin{cases} 1 - \frac{ x }{a} & \text{if } x < a \\ 0 & \text{if } x > a \end{cases} \quad a > 0$		$2\sqrt{\frac{2}{\pi}} \frac{\sin^2(\frac{a\omega}{2})}{a\omega^2}$
4.	$\begin{cases} x & \text{if } x < a \\ 0 & \text{if } x > a \end{cases} \quad a > 0$		$i\sqrt{\frac{2}{\pi}} \frac{a\omega \cos(a\omega) - \sin(a\omega)}{\omega^2}$
5.	$\begin{cases} \sin x & \text{if } x < \pi \\ 0 & \text{if } x > \pi \end{cases}$		$i\sqrt{\frac{2}{\pi}} \frac{\sin(\pi\omega)}{\omega^2 - 1}$
6.	$\begin{cases} \sin(ax) & \text{if } x < b \\ 0 & \text{if } x > b \end{cases} \quad a, b > 0$		$i\sqrt{\frac{2}{\pi}} \frac{\omega \cos(b\omega) \sin(ab) - a \cos(ab) \sin(b\omega)}{\omega^2 - a^2}$
7.	$\frac{1}{a^2 + x^2}, a > 0$		$\sqrt{\frac{\pi}{2}} \frac{e^{-a \omega }}{a}$
8.	$\frac{x}{a^2 + x^2}, a > 0$		$-i\sqrt{\frac{\pi}{2}} \operatorname{sgn} \omega e^{-a \omega }$
9.	$\sqrt{\frac{2}{\pi}} \frac{a}{1 + a^2 x^2}, a > 0$		$e^{-\frac{ \omega }{a}}$
10.	$\frac{\sin ax}{x}, a > 0$		$\begin{cases} \sqrt{\frac{\pi}{2}} & \text{if } \omega < a \\ \frac{1}{2} \sqrt{\frac{\pi}{2}} & \text{if } \omega = a \\ 0 & \text{if } \omega > a \end{cases}$
11.	$\frac{4}{\sqrt{2\pi}} \frac{\sin^2(\frac{1}{2}ax)}{ax^2}, a > 0$		$\begin{cases} 1 - \frac{ \omega }{a} & \text{if } \omega < a \\ 0 & \text{if } \omega > a \end{cases}$
12.	$\frac{4}{\sqrt{2\pi}} \frac{\sin^2(ax) - \sin^2(\frac{1}{2}ax)}{ax^2}, a > 0$		$\begin{cases} 1 & \text{if } x < a \\ (-x + 2a)/a & \text{if } a < x < 2a \\ (x + 2a)/a & \text{if } a < x < 2a \\ 0 & \text{if } x > 2a \end{cases}$
13.	$e^{-a x }, a > 0$		$\sqrt{\frac{2}{\pi}} \frac{a}{a^2 + \omega^2}$
14.	$\begin{cases} e^{-ax} & \text{if } x > 0 \\ 0 & \text{if } x < 0 \end{cases}, a > 0$		$\frac{1}{\sqrt{2\pi}} \frac{1}{a + i\omega}$
15.	$\begin{cases} 0 & \text{if } x > 0 \\ e^{ax} & \text{if } x < 0 \end{cases}, a > 0$		$\frac{1}{\sqrt{2\pi}} \frac{1}{a - i\omega}$
16.	$ x ^n e^{-a x }, a > 0, n > 0$		$\frac{\Gamma(n+1)}{\sqrt{2\pi}} \left(\frac{1}{(a - i\omega)^{1+n}} + \frac{1}{(a + i\omega)^{1+n}} \right)$

Table of Fourier Transforms (continued)

$f(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} \hat{f}(\omega) e^{ix\omega} d\omega$	$\hat{f}(\omega) = \mathcal{F}(f)(\omega) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} f(x) e^{-i\omega x} dx$
17. $e^{-\frac{a}{2}x^2}, a > 0$	$\frac{1}{\sqrt{a}} e^{-\frac{\omega^2}{2a}}$
18. $e^{-ax^2}, a > 0$	$\frac{1}{\sqrt{2a}} e^{-\frac{\omega^2}{4a}}$
19. $xe^{-\frac{a}{2}x^2}, a > 0$	$\frac{-i\omega}{a^{3/2}} e^{-\frac{\omega^2}{2a}}$
20. $x^2 e^{-\frac{a}{2}x^2}, a > 0$	$\frac{a - \omega^2}{a^{5/2}} e^{-\frac{\omega^2}{2a}}$
21. $x^3 e^{-\frac{a}{2}x^2}, a > 0$	$\frac{-i\omega(3a - \omega^2)}{a^{7/2}} e^{-\frac{\omega^2}{2a}}$
22. $e^{-\frac{x^2}{2}} H_n(x),$ H_n, n th Hermite polynomial	$(-1)^n i^n e^{-\frac{\omega^2}{2}} H_n(\omega)$
23. $J_0(x),$ Bessel function of order 0	$\begin{cases} \sqrt{\frac{2}{\pi}} \frac{1}{\sqrt{1-\omega^2}} & \text{if } \omega < 1 \\ 0 & \text{if } \omega > 1 \end{cases}$
24. $J_n(x),$ Bessel function of order $n \geq 0$	$\begin{cases} \sqrt{\frac{2}{\pi}} \frac{(-i)^n}{\sqrt{1-\omega^2}} T_n(\omega) & \text{if } \omega < 1 \\ 0 & \text{if } \omega > 1 \end{cases}$
$T_n, \text{Chebyshev polynomial of degree } n.$	
Special Transforms	
25. $\mathcal{F}(\delta_0(x))(\omega) = \frac{1}{\sqrt{2\pi}}$	27. $\mathcal{F}\left(\sqrt{\frac{2}{\pi}} \frac{1}{x}\right)(\omega) = -i \operatorname{sgn} \omega$
26. $\mathcal{F}(\delta_0(x-a))(\omega) = \frac{1}{\sqrt{2\pi}} e^{-ia\omega}$	28. $\mathcal{F}(e^{iax})(\omega) = \sqrt{2\pi} \delta_0(\omega - a)$
Operational Properties	
29. $\mathcal{F}(af + bg)(\omega) = a\mathcal{F}(f) + b\mathcal{F}(g)$	36. $\mathcal{F}(fg)(\omega) = \mathcal{F}(f) * \mathcal{F}(g)(\omega)$
30. $\mathcal{F}(f')(\omega) = i\omega\mathcal{F}(f)(\omega)$	37. $\mathcal{F}(f(x-a))(\omega) = e^{-ia\omega}\mathcal{F}(f)(\omega)$
31. $\mathcal{F}(f'')(\omega) = -\omega^2\mathcal{F}(f)(\omega)$	38. $\mathcal{F}(e^{iax}f(x))(\omega) = \mathcal{F}(f)(\omega - a)$
32. $\mathcal{F}(f^{(n)})(\omega) = (i\omega)^n\mathcal{F}(f)(\omega)$	39. $\mathcal{F}(\cos(ax)f(x))(\omega) = \frac{\mathcal{F}(f)(\omega-a) + \mathcal{F}(f)(\omega+a)}{2}$
33. $\mathcal{F}(xf(x))(\omega) = i \frac{d}{d\omega} \mathcal{F}(f)(\omega)$	40. $\mathcal{F}(\sin(ax)f(x))(\omega) = \frac{\mathcal{F}(f)(\omega-a) - \mathcal{F}(f)(\omega+a)}{2i}$
34. $\mathcal{F}(x^n f(x))(\omega) = i^n \frac{d^n}{d\omega^n} \mathcal{F}(f)(\omega)$	41. $\mathcal{F}(f(ax))(\omega) = \frac{1}{ a } \mathcal{F}(f)\left(\frac{\omega}{a}\right), a \neq 0$
35. $\mathcal{F}(f * g)(\omega) = \mathcal{F}(f)(\omega)\mathcal{F}(g)(\omega)$	42. $f(x) = \mathcal{F}(\hat{f})(-x), \mathcal{F}(\mathcal{F}(f)) = f(-x)$

Table of Fourier Cosine Transforms

$f(x) = \sqrt{\frac{2}{\pi}} \int_0^{\infty} \mathcal{F}_c(f)(\omega) \cos \omega x \, d\omega,$ $0 < x < \infty$		$\mathcal{F}_c(f)(\omega) = \hat{f}_c(\omega) = \sqrt{\frac{2}{\pi}} \int_0^{\infty} f(x) \cos \omega x \, dx,$ $0 \leq \omega < \infty$	
1.	$\begin{cases} 1 & \text{if } 0 < x < a \\ 0 & \text{otherwise} \end{cases}$		$\sqrt{\frac{2}{\pi}} \frac{\sin a\omega}{\omega}$
2.	$e^{-ax}, \quad a > 0$		$\sqrt{\frac{2}{\pi}} \frac{a}{a^2 + \omega^2}$
3.	$x e^{-ax}, \quad a > 0$		$\sqrt{\frac{2}{\pi}} \frac{a^2 - \omega^2}{(a^2 + \omega^2)^2}$
4.	$e^{-ax^2/2}, \quad a > 0$		$\frac{1}{\sqrt{a}} e^{-\omega^2/2a}$
5.	$\cos ax e^{-ax}, \quad a > 0$		$\sqrt{\frac{2}{\pi}} \frac{a\omega^2 + 2a^3}{4a^4 + \omega^4}$
6.	$\sin ax e^{-ax}, \quad a > 0$		$\sqrt{\frac{2}{\pi}} \frac{2a^3 - a\omega^2}{4a^4 + \omega^4}$
7.	$\frac{a}{a^2 + x^2}, \quad a > 0$		$\sqrt{\frac{\pi}{2}} e^{-a\omega}$
8.	$x^p, \quad 0 < p < 1$		$\sqrt{\frac{2}{\pi}} \frac{\Gamma(p) \cos(p\omega/2)}{\omega^p}$
9.	$\begin{cases} \cos x & \text{if } 0 < x < a \\ 0 & \text{otherwise} \end{cases}$		$\frac{1}{\sqrt{2\pi}} \left[\frac{\sin a(1-\omega)}{1-\omega} + \frac{\sin a(1+\omega)}{1+\omega} \right]$
Operational Properties			
10.	$\alpha f(x) + \beta g(x)$		$\alpha \mathcal{F}_c(f)(\omega) + \beta \mathcal{F}_c(g)(\omega)$
11.	$f(ax), \quad a > 0$		$\frac{1}{a} \hat{f}_c\left(\frac{\omega}{a}\right)$
12.	$f'(x)$		$\omega \hat{f}_s(\omega) - \sqrt{\frac{2}{\pi}} f(0)$
13.	$f''(x)$		$-\omega^2 \hat{f}_c(\omega) - \sqrt{\frac{2}{\pi}} f'(0)$
14.	$xf(x)$		$[\hat{f}_s]'(\omega)$
15.	$\mathcal{F}_c(\mathcal{F}_c f)$		f

Table of Fourier Sine Transforms

$$f(x) = \sqrt{\frac{2}{\pi}} \int_0^{\infty} \mathcal{F}_s(f)(\omega) \sin \omega x \, d\omega,$$

$$\mathcal{F}_s(f)(\omega) = \hat{f}_s(\omega) = \sqrt{\frac{2}{\pi}} \int_0^{\infty} f(x) \sin \omega x \, dx,$$

$$0 < x < \infty$$

$$0 < \omega < \infty$$

1.	$\begin{cases} 1 & \text{if } 0 < x < a \\ 0 & \text{otherwise} \end{cases}$	$\sqrt{\frac{2}{\pi}} \frac{1 - \cos a\omega}{\omega}$
2.	$e^{-ax}, \quad a > 0$	$\sqrt{\frac{2}{\pi}} \frac{\omega}{a^2 + \omega^2}$
3.	$x e^{-ax}, \quad a > 0$	$\sqrt{\frac{2}{\pi}} \frac{2a\omega}{(a^2 + \omega^2)^2}$
4.	$\frac{e^{-ax}}{x}, \quad a > 0$	$\sqrt{\frac{2}{\pi}} \tan^{-1} \frac{\omega}{a}$
5.	$\frac{1}{2} x e^{-ax^2}, \quad a > 0$	$\frac{\omega}{a^{3/2}} e^{-\omega^2/2a}$
6.	$\cos ax e^{-ax}, \quad a > 0$	$\sqrt{\frac{2}{\pi}} \frac{\omega^3}{4a^4 + \omega^4}$
7.	$\sin ax e^{-ax}, \quad a > 0$	$\sqrt{\frac{2}{\pi}} \frac{2a^2\omega}{4a^4 + \omega^4}$
8.	$\frac{x}{a^2 + x^2}, \quad a > 0$	$\sqrt{\frac{\pi}{2}} e^{-a\omega}$
9.	$x^{p-1}, \quad 0 < p < 1$	$\sqrt{\frac{2}{\pi}} \frac{\Gamma(p) \cos(\pi p/2)}{\omega^p}$
10.	$\begin{cases} \sin x & \text{if } 0 < x < a \\ 0 & \text{otherwise} \end{cases}$	$\frac{1}{\sqrt{2\pi}} \left[\frac{\sin a(1-\omega)}{1-\omega} - \frac{\sin a(1+\omega)}{1+\omega} \right]$

Operational Properties

11.	$\alpha f(x) + \beta g(x)$	$\alpha \mathcal{F}_s(f)(\omega) + \beta \mathcal{F}_s(g)(\omega)$
12.	$f(ax), \quad a > 0$	$\frac{1}{a} \hat{f}_s\left(\frac{\omega}{a}\right)$
13.	$f'(x)$	$-\omega \hat{f}_c(\omega)$
14.	$f''(x)$	$-\omega^2 \hat{f}_s(\omega) + \sqrt{\frac{2}{\pi}} \omega f(0)$
15.	$xf(x)$	$-\left[\hat{f}_c\right]'(\omega)$
16.	$\mathcal{F}_s(\mathcal{F}_s f)$	f

Table of Laplace Transforms

$f(t), t \geq 0$	$F(s) = \mathcal{L}(f)(s) = \int_0^{\infty} f(t)e^{-st} dt,$
1. 1	$\frac{1}{s}, s > 0$
2. $t^n, n = 1, 2, \dots$	$\frac{n!}{s^{n+1}}, s > 0$
3. $t^a (a > -1)$	$\frac{\Gamma(a+1)}{s^{a+1}}, s > 0$
4. e^{at}	$\frac{1}{s-a}, s > a$
5. $t^n e^{at}$	$\frac{n!}{(s-a)^{n+1}}, s > a$
6. $\frac{e^{at} - e^{bt}}{a-b}$	$\frac{1}{(s-a)(s-b)}, s > \max(a, b)$
7. $\frac{ae^{at} - be^{bt}}{a-b}$	$\frac{s}{(s-a)(s-b)}, s > \max(a, b)$
8. $\sin kt$	$\frac{k}{s^2+k^2}, s > 0$
9. $\cos kt$	$\frac{s}{s^2+k^2}, s > 0$
10. $e^{at} \sin kt$	$\frac{k}{(s-a)^2+k^2}, s > a$
11. $e^{at} \cos kt$	$\frac{s-a}{(s-a)^2+k^2}, s > a$
12. $t \sin kt$	$\frac{2ks}{(s^2+k^2)^2}, s > 0$
13. $t \cos kt$	$\frac{s^2-k^2}{(s^2+k^2)^2}, s > 0$
14. $\frac{1}{2a^3}(\sin at - at \cos at)$	$\frac{1}{(s^2+a^2)^2}, s > 0$
15. $\sinh kt$	$\frac{k}{s^2-k^2}, s > k $
16. $\cosh kt$	$\frac{s}{s^2-k^2}, s > k $
17. $e^{at} \sinh kt$	$\frac{k}{(s-a)^2-k^2}, s > a + k $
18. $e^{at} \cosh kt$	$\frac{s-a}{(s-a)^2-k^2}, s > a + k $
19. $t \sinh kt$	$\frac{2ks}{(s^2-k^2)^2}, s > k $
20. $t \cosh kt$	$\frac{s^2+k^2}{(s^2-k^2)^2}, s > k $
21. $\frac{1}{2k^3}(kt \cosh kt - \sinh kt)$	$\frac{1}{(s^2-k^2)^2}, s > k $

Table of Laplace Transforms (continued)

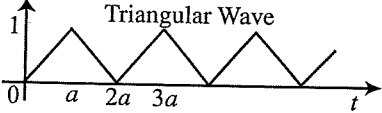
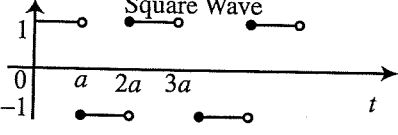
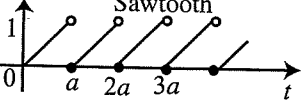
$f(t), t \geq 0$	$F(s) = \mathcal{L}(f)(s) = \int_0^\infty f(t)e^{-st} dt,$
22. $\delta_0(t-t_0), t_0 \geq 0$	$e^{-t_0s}, s > 0$
23. $\mathcal{U}_0(t-a) = \begin{cases} 0 & \text{if } t < a \\ 1 & \text{if } t \geq a \end{cases} \quad (a > 0)$	$\frac{e^{-as}}{s}, s > 0$
24. $f(t+T) = f(t) \quad (T > 0)$	$\frac{1}{1-e^{-Ts}} \int_0^T e^{-st} f(t) dt$
25. $f(t+T) = -f(t) \quad (T > 0)$	$\frac{1}{1+e^{-Ts}} \int_0^T e^{-st} f(t) dt$
26. 	$\frac{1}{as^2} \left[\frac{1-e^{-as}}{1+e^{-as}} \right] = \frac{1}{as^2} \tanh\left(\frac{as}{2}\right), s > 0$
27. 	$\frac{1}{s} \left[\frac{1-e^{-as}}{1+e^{-as}} \right] = \frac{1}{s} \tanh\left(\frac{as}{2}\right), s > 0$
28. 	$\frac{1}{as^2} - \frac{e^{-as}}{s(1-e^{-as})}, s > 0$
29. $\frac{\sin at}{t}$	$\tan^{-1}\left(\frac{a}{s}\right), s > 0$
30. $J_0(at)$	$\frac{1}{\sqrt{s^2+a^2}}, s > 0$
31. $J_0(a\sqrt{t})$	$\frac{e^{-a^2/4s}}{s}, s > 0$
32. $t^p J_p(at) \quad (p > -\frac{1}{2})$	$\frac{2^p a^p \Gamma(p+\frac{1}{2})}{\sqrt{\pi}(s^2+a^2)^{p+\frac{1}{2}}}, s > 0$
33. $\frac{\sqrt{\pi}}{\Gamma(k)} \left(\frac{t}{2a}\right)^{k-\frac{1}{2}} J_{k-\frac{1}{2}}(at) \quad (k > 0)$	$\frac{1}{(s^2+a^2)^k}, s > 0$
34. $\frac{\sqrt{\pi}}{\Gamma(k)} a \left(\frac{t}{2a}\right)^{k-\frac{1}{2}} J_{k-\frac{3}{2}}(at) \quad (k > \frac{1}{2})$	$\frac{s}{(s^2+a^2)^k}, s > 0$
35. $2 \sum_{m=1}^n \binom{2n-m-1}{n-1} \frac{t^{m-1} \cos(at - \frac{m\pi}{2})}{(2a)^{2n-m} (m-1)!}$ (n an integer ≥ 1)	$\frac{1}{(s^2+a^2)^n}, s > 0$
36. $\frac{1}{(n-1)!} \sum_{m=1}^{n-1} \frac{(2n-m-3)!}{(m-1)!(n-m-1)!} \frac{t^m \cos(at - \frac{m\pi}{2})}{(2a)^{2n-m-2}}$ (n an integer ≥ 2)	$\frac{s}{(s^2+a^2)^n}, s > 0$

Table of Laplace Transforms (continued)

$f(t), t \geq 0$	$F(s) = \mathcal{L}(f)(s) = \int_0^{\infty} f(t)e^{-st} dt,$
37. $\operatorname{erf}(at) \quad (a > 0)$	$\frac{1}{s} e^{s^2/4a^2} \operatorname{erfc}\left(\frac{s}{2a}\right), \quad s > 0$
38. $\operatorname{erf}(a\sqrt{t})$	$\frac{a}{s\sqrt{s+a^2}}, \quad s > 0$
39. $e^{-a^2 t^2} \quad (a > 0)$	$\frac{\sqrt{\pi}}{2a} e^{s^2/4a^2} \operatorname{erfc}\left(\frac{s}{2a}\right), \quad s > 0$
40. $\frac{1}{\sqrt{\pi t}} e^{-a^2/4t} \quad (a \geq 0)$	$\frac{e^{-a\sqrt{s}}}{\sqrt{s}}, \quad s > 0$
41. $\frac{a}{2\sqrt{\pi t^3/2}} e^{-a^2/4t} \quad (a > 0)$	$e^{-a\sqrt{s}}, \quad s > 0$
42. $\operatorname{erfc}\left(\frac{a}{2\sqrt{t}}\right) \quad (a \geq 0)$	$\frac{1}{s} e^{-a\sqrt{s}}, \quad s > 0$
Operational Properties	
43. $\alpha f(t) + \beta g(t)$	$\alpha F(s) + \beta G(s)$
44. $f'(t)$	$sF(s) - f(0)$
45. $f''(t)$	$s^2 F(s) - sf(0) - f'(0)$
46. $f^{(n)}(t)$	$s^n F(s) - s^{n-1} f(0) - \dots - f^{(n-1)}(0)$
47. $-tf(t)$	$F'(s)$
48. $t^n f(t)$	$(-1)^n F^{(n)}(s)$
49. $\int_0^t f(\tau) d\tau$	$\frac{1}{s} F(s), \quad s > 0$
50. $\int_0^t \int_0^\tau f(\rho) d\rho d\tau$	$\frac{1}{s^2} F(s), \quad s > 0$
51. $\frac{f(t)}{t}$	$\int_s^{\infty} F(u) du$
52. $\frac{f(t)}{t^2}$	$\int_s^{\infty} \int_\sigma^{\infty} F(u) dud\sigma$
53. $\mathcal{U}(t-a)f(t-a) \quad (a > 0)$	$e^{-as} F(s)$
54. $e^{at} f(t)$	$F(s-a)$
55. $f(ct) \quad (c > 0)$	$\frac{1}{c} F\left(\frac{s}{c}\right)$
56. $f * g(t) = \int_0^t f(\tau)g(t-\tau) d\tau$	$F(s)G(s)$