MATEMATISKA INSTITUTIONEN
KTH
Kursledare: Svante Linusson
telefon 08-790 94 44

Hand in problems in SF2715 Applied Combinatorics VT2011, set 1
Correct solutions to the following problems will give bonus points on the final exam.

Be sure to write solutions with clear arguments that are easy to follow. You should try to have a level of details so your solution would be understandable to other students. Staple your papers together in the top left corner and write down your solutions in order. Write your name in the top right corner.

Hederskodex (Code of conduct): It is assumed that:
-you shall solve the problems on your own and write down your own solution
-if you in spite of this are using something you have gotten from somewhere else for some reason (a friend, a book or the internet etc.) you should give a reference to the source.

Your solutions to the problems are due April 7th before class starts.

PLEASE: Motivate your solutions clearly!

1. Simplify
$$\sum_{d=0}^{n}\left(\binom{d}{k}\cdot(d+1)-\binom{d+1}{k+1}\cdot k\right)$$
as far as you can. It should be an expression involving only one binomial coefficient.

   Example:
   If $n=4, k=2$, then $\sum_{d=0}^{4}\left(\binom{d}{2}\cdot(d+1)-\binom{d+1}{3}\cdot 2\right)=15$ which could be $\binom{6}{2}$ or $\binom{6}{4}$.

2. Let a_n be the number of words over the alphabet $Q=\{0,1,2\}$ with forbidden subwords $X=\{00,12,21\}$.
   (i) Use Theorem 2.2 i the handout, part 1, to find a recursion for the numbers a_n.
   (ii) Find the ordinary generating function $A(x)=\sum_{n\geq 0} a_n x^n$.
   (iii) Find an explicit expression for a_n using $A(x)$.
   (iv) Can you find a combinatorial explanation for the formula for a_n?

3. You want to count words of length n from the alphabet $\{a,b,c\}$ with the property that there must be an even number of a:s and at least 2 c:s. Determine an exact formula.

Lycka till!

Svante