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Hand in problems in SF2715 Applied Combinatorics VT2011, set 2
Correct solutions to the following problems will give bonus points on the final exam.

Be sure to write solutions with clear arguments that are easy to follow. You should try to have a level of details so your solution would be understandable to other students. Staple your papers together in the top left corner and write down your solutions in order. Write your name in the top right corner.

Hederskodex (Code of conduct): It is assumed that:
-you shall solve the problems on your own and write down your own solution
-if you in spite of this are using something you have gotten from somewhere else for some reason (a friend, a book or the internet etc.) you should give a reference to the source.

Your solutions to the problems are due April 29 before class starts.

PLEASE: Motivate your solutions clearly!

1. Let $a(n, k)$ be the number of set partitions of $\{1, 2, \dots, n\}$ such that n is in a block of size at least two, i.e. n must not be in a block of size 1.
   Example: $a(3, 2) = 2$, since $\{1\}, \{2, 3\}$ and $\{2\}, \{1, 3\}$ are such partitions, whereas $\{1, 2\}, \{3\}$ is not such a partition.

   Write a formula for $a(n, k)$ in terms of the Stirling numbers of the second kind $S(i, j)$.

2. Use generating functions to prove the following statement about integer partitions for any integer $n \geq 1$:
   The number of partitions of n where a part of size k can occur at most k times
   is equal to
   the number of partitions of n where parts of the form $k(k + 1)$ are not used.

   Example: Let $n = 5$. Partitions of the first kind are $5, 4+1, 3+2$ and $2+2+1$ (but not $2+1+1+1$ or $1+1+1+1+1$). Partitions of the second kind are $5, 4+1, 3+1+1$ and $1+1+1+1+1$, since we may not use parts of the form $k(k+1)$ which are $2, 6, 12, \dots$.

3. Let $\pi = 5162374$ or written in cycle form $(1, 5, 3, 6, 7, 4, 2)$. For each of the seven permutations $\pi, \pi^2, \pi^3, \pi^4, \pi^5.\pi^6$ and π^7 determine the pair of tableaux using the RSK-correspondance.

   You may use theorems in the book to facilitate your computations.

Lycka till!

Svante