MATEMATISKA INSTITUTIONEN
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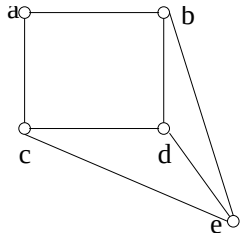
Hand in problems in SF2715 Applied Combinatorics VT2011, set 3
Correct solutions to the following three problems will give bonus points on the final exam.

Be sure to write solutions with clear arguments that are easy to follow. You should try to have a level of details so your solution would be understandable to other students. Staple your papers together in the top left corner and write down your solutions in order. Write your name in the top right corner.

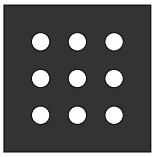
Hederskodex (Code of conduct): It is assumed that:
-you shall solve the problems on your own and write down your own solution
-if you in spite of this are using something you have gotten from somewhere else for some reason (a friend, a book or the internet etc.) you should give a reference to the source.

Your solutions to the problems are due May 17 before class starts.

1. Use the Matrix-Tree Theorem to compute the number of spanning trees in the following graph.



2. Consider a candle holder with holes of the following form with room for nine candles.



   We consider two placements of candles as equal if we may obtain one from the other by rotation of the candle holder. For every integer $k \geq 0$ determine how many different ways there are to place k identical candles in the candle holder.

3. Let a network $G = (V, E)$ be given with source s, target t and capacities $c(x, y) \geq 0$ on every (directed) edge $(x, y) \in E$. A *potential* α is a real valued function on V such that

   i) $\alpha(s) = 0$ and

   ii) $\alpha(y) - \alpha(x) \leq c(x, y)$ for every $(x, y) \in E$.

Define the "length" of a directed (s,t)-path P to be $\ell(P) := \sum_{e \in E(P)} c(e)$. Prove that

$$\min_P \ell(P) = \max_\alpha \alpha(t).$$

Lycka till!

Svante