10.3.14.

$$\mathbf{X}' = \begin{pmatrix} x \\ y \end{pmatrix}' = \begin{pmatrix} 2x - y^2 \\ -y + xy \end{pmatrix} = \mathbf{g}(\mathbf{X})$$

$$\text{Kritiska punkter: } \begin{pmatrix} 0 \\ 0 \end{pmatrix} = \begin{pmatrix} x \\ y \end{pmatrix}' = \begin{pmatrix} 2x - y^2 \\ -y + xy \end{pmatrix}.$$

$$\begin{cases} 2x - y^2 = 0 \\ -y + xy = -y(1 - x) = 0 \end{cases}$$

$$a)\quad y = 0 \qquad x = 0\ ,\ (x, y) = (0,0)$$

$$b)\quad x = 1 \qquad y = \pm\sqrt{2}\ ,\ (x, y) = (1, \pm\sqrt{2})$$

$$\mathbf{g}'(\mathbf{X}) = \begin{pmatrix} 2 & -2y \\ y & -1+x \end{pmatrix}$$

$$\mathbf{g}'(0,0) = \begin{pmatrix} 2 & 0 \\ 0 & -1 \end{pmatrix}, \quad \lambda_1 = 2, \quad \lambda_2 = -1.$$

Olika tecken på λ.

(0,0) är en sadelpunkt.

$$\mathbf{g}'(1,\sqrt{2}) = \begin{pmatrix} 2 & -2\sqrt{2} \\ \sqrt{2} & 0 \end{pmatrix}$$

$$0 = \lambda^2 - 2\lambda + 4 = (\lambda - 1)^2 + 3$$

$(1,\sqrt{2})$ är en instabil spiral.

$$\mathbf{g}'(1, -\sqrt{2}) = \begin{pmatrix} 2 & 2\sqrt{2} \\ -\sqrt{2} & 0 \end{pmatrix}$$

$$0 = \lambda^2 - 2\lambda + 4 = (\lambda - 1)^2 + 3$$

$(1, -\sqrt{2})$ är en instabil spiral.

Alternativ framställning.

$$(0,0)$$

$$\begin{pmatrix} x \\ y \end{pmatrix}' = \begin{pmatrix} 2x \\ -y \end{pmatrix} + \begin{pmatrix} -y^2 \\ xy \end{pmatrix} = \begin{pmatrix} 2 & 0 \\ 0 & -1 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} + \begin{pmatrix} -y^2 \\ xy \end{pmatrix}$$

$$\mathbf{A} = \begin{pmatrix} 2 & 0 \\ 0 & -1 \end{pmatrix}, \quad \mathbf{0} = (\mathbf{A} - \lambda \mathbf{I})\mathbf{v} = \begin{pmatrix} 2-\lambda & 0 \\ 0 & -1-\lambda \end{pmatrix}\mathbf{v}$$

$$\lambda_1 = 2\ ,\ \mathbf{v}_1 = \begin{pmatrix} 1 \\ 0 \end{pmatrix},\ \lambda_2 = -1\ ,\ \mathbf{v}_2 = \begin{pmatrix} 0 \\ 1 \end{pmatrix}.$$

$$(1, \sqrt{2})$$

$$\text{Sätt:} \quad \begin{cases} u = x - 1 \\ v = y - \sqrt{2} \end{cases} \quad \begin{cases} u' = x' \\ v' = y' \end{cases}.$$

$$\begin{pmatrix} u' \\ v' \end{pmatrix} = \begin{pmatrix} 2(u+1) - (v+\sqrt{2})^2 \\ -(v+\sqrt{2}) + (u+1)(v+\sqrt{2}) \end{pmatrix}$$

$$\begin{pmatrix} u' \\ v' \end{pmatrix} = \begin{pmatrix} 2u - v^2 - 2v\sqrt{2} \\ uv + u\sqrt{2} \end{pmatrix}$$

$$\begin{pmatrix} u' \\ v' \end{pmatrix} = \begin{pmatrix} 2 & -2\sqrt{2} \\ \sqrt{2} & 0 \end{pmatrix} \begin{pmatrix} u \\ v \end{pmatrix} + \begin{pmatrix} -v^2 \\ uv \end{pmatrix}$$

$$\mathbf{B} = \begin{pmatrix} 2 & -2\sqrt{2} \\ \sqrt{2} & 0 \end{pmatrix}$$

$$0 = \det(\mathbf{B} - \lambda \mathbf{I}) = \lambda^2 - 2\lambda + 4 = (\lambda - 1)^2 + 3.$$

$$\lambda = 1 \pm i\sqrt{3}\ ,\ \ \text{Instabil spiral.}$$

$$(1,-\sqrt{2})$$

Sätt: $\begin{cases} u = x-1 \\ v = y+\sqrt{2} \end{cases} \quad \begin{cases} u' = x' \\ v' = y' \end{cases}$.

$$\begin{pmatrix} u' \\ v' \end{pmatrix} = \begin{pmatrix} 2(u+1)-(v-\sqrt{2})^2 \\ -(v-\sqrt{2})+(u+1)(v-\sqrt{2}) \end{pmatrix}$$

$$\begin{pmatrix} u' \\ v' \end{pmatrix} = \begin{pmatrix} 2u-v^2+2v\sqrt{2} \\ uv-u\sqrt{2} \end{pmatrix}$$

$$\begin{pmatrix} u' \\ v' \end{pmatrix} = \begin{pmatrix} 2u + 2v\sqrt{2} \\ -u\sqrt{2} \end{pmatrix} + \begin{pmatrix} -v^2 \\ uv \end{pmatrix}$$

$$\begin{pmatrix} u' \\ v' \end{pmatrix} = \begin{pmatrix} 2 & 2\sqrt{2} \\ -\sqrt{2} & 0 \end{pmatrix}\begin{pmatrix} u \\ v \end{pmatrix} + \begin{pmatrix} -v^2 \\ uv \end{pmatrix}$$

$$\mathbf{C} = \begin{pmatrix} 2 & 2\sqrt{2} \\ -\sqrt{2} & 0 \end{pmatrix}$$

$$0 = \det(\mathbf{C} - \lambda \mathbf{I}) = \lambda^2 - 2\lambda + 4 = (\lambda - 1)^2 + 3.$$

$$\lambda = 1 \pm i\sqrt{3}\ ,\ \text{Instabil spiral.}$$