10.3.33.

$$\mathbf{X}' = \begin{pmatrix} x \\ y \end{pmatrix}' = \begin{pmatrix} 2xy \\ 1 - x^2 + y^2 \end{pmatrix} = \mathbf{g}(\mathbf{X})$$

Kritiska punkter: $\begin{pmatrix} 0 \\ 0 \end{pmatrix} = \begin{pmatrix} x \\ y \end{pmatrix}' = \begin{pmatrix} 2xy \\ 1 - x^2 + y^2 \end{pmatrix}.$

$a)$ $x = 0$ $\quad 1 + y^2 = 0$, Reell lösning saknas.

$b)$ $y = 0$ $\quad x = \pm 1$, $(x, y) = (\pm 1, 0)$

$$\mathbf{g}'(\mathbf{X}) = \begin{pmatrix} 2y & 2x \\ -2x & 2y \end{pmatrix}$$

$$\mathbf{g}'(1,0) = \begin{pmatrix} 0 & 2 \\ -2 & 0 \end{pmatrix}, \lambda = \pm 2i.$$

Ingen slutsats kan dras.

$$\mathbf{g}'(-1,0) = \begin{pmatrix} 0 & -2 \\ 2 & 0 \end{pmatrix}, \lambda = \pm 2i.$$

Ingen slutsats kan dras.

Vi använder fasplanmetoden.

$$\frac{\mathrm{dy}}{\mathrm{dx}} = \frac{\dot{y}}{\dot{x}} = \frac{1 - x^2 + y^2}{2xy}$$

$$2xy\frac{\mathrm{dy}}{\mathrm{dx}} = 1 - x^2 + y^2$$

Sätt: $z = y^2, \quad \frac{\mathrm{dz}}{\mathrm{dx}} = 2y\frac{\mathrm{dy}}{\mathrm{dx}}$

$$x\frac{\mathrm{dz}}{\mathrm{dx}} = 1 - x^2 + z$$

$$\frac{dz}{dx} - \frac{1}{x} z = \frac{1}{x} - x$$

$$\frac{1}{x}\frac{dz}{dx} - \frac{1}{x}\frac{1}{x} z = \frac{1}{x}\left(\frac{1}{x} - x\right)$$

$$\frac{d}{dx}\left\{\frac{1}{x} z\right\} = \frac{1}{x^2} - 1$$

$$\frac{z}{x} = -\frac{1}{x} - x + C$$

$$z = -1 - x^2 + Cx$$

$$y^2 = -1 - x^2 + Cx$$

$$x^2 - Cx + y^2 = -1, \; C = 2a$$

$$(x - a)^2 + y^2 = a^2 - 1$$

Cirklar.