DIFFERENTIAL GEOMETRY, FALL 2011
GRADUATE LEVEL, ADDITIONAL HOMEWORK 4
SEMI-RIEMANNIAN GEOMETRY

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Solve and hand in your solution to the following problem on Semi-Riemannian geometry (Chapter 13). Please pay attention to the presentation as well as the arguments you give in your solutions.

Problem 1. (The Schwarzschild metric.) Let $\mathbb{R}^3 \setminus \{0\}$ have spherical coordinates $0 < r < \infty$, $0 \leq \theta < \pi$, $0 \leq \varphi < 2\pi$, so that the standard flat metric on $\mathbb{R}^3$ is $dr^2 + r^2(d\theta^2 + \sin^2\theta d\varphi^2)$. Equip $\mathbb{R} \times (\mathbb{R}^3 \setminus \{r \leq 2m\})$ with the Lorentzian metric

$$g = -\left(1 - \frac{2m}{r}\right) dt^2 + \left(1 - \frac{2m}{r}\right)^{-1} dr^2 + r^2(d\theta^2 + \sin^2\theta d\varphi^2)$$

where m is a positive constant. Show that g is a solution to the vacuum Einstein equations

$$\operatorname{Ric} - \frac{1}{2} S g = 0 \quad \Longleftrightarrow \quad \operatorname{Ric} = 0$$

where the scalar curvature $S = \operatorname{tr_g} \operatorname{Ric}$ is the trace of the Ricci curvature.

Date: December 8, 2011.