SF2735 Homological algebra and algebraic topology
Exercise set 4

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The solutions are to be handed in no later than Monday, 19th of December. Please pay attention to the presentation as well as the arguments given in the solutions. In the solutions you can only use the material given in the notes or during the lecture.

Exercise

(a). (**2 points**) Let $\alpha : \Delta^1 \to X$ be a 1-dimensional singular simplex in X. Define $\beta : \Delta^1 \to X$ by the formula $\beta(t_0, t_1) := \alpha(t_1, t_0)$. Show that $\alpha + \beta$ is a boundary in $S_1(X)$.

(b). (**3 points**) Let $\sigma : \Delta^1 \to S^1$ be the map given by $\sigma(t_0, t_1) := (\sin(2t_0\pi), \cos(2t_0\pi))$. Show that this singular simplex is a cycle. Show moreover that its homology class is a generator of $H_1(S^1) = \mathbf{Z}$.

(c). (**2 points**) Let $\tau : \Delta^1 \to S^1$ be the map given by $\tau(t_0, t_1) := (\sin(2t_1\pi), \cos(2t_1\pi))$. Show that this singular simplex is a cycle. Show moreover that $\sigma + \tau$ is a boundary, where σ is the singular simplex defined in part b. Conclude that in $H_1(S^1)$, we have $[\tau] = -[\sigma]$.

(d). (**1 point**) Use parts b and c to show that the map $f : S^1 \to S^1$, given by $f(x_1, x_2) := (x_1, -x_2)$, induces multiplication by -1 on $H_1(S^1)$.

(e). (**2 points**) Let $\tau : \Delta^1 \to S^1$ be the map given by $\tau(t_0, t_1) := (\sin(4t_0\pi), \cos(4t_0\pi))$. Show that this singular simplex is a cycle and that its homology class $[\tau] \in H_1(S^1)$ is equal to $2[\sigma]$, where σ is the singular simplex defined in part b.