Matematiska Institutionen
KTH

Homework number 1 to SF2736, fall 2011.

Please, deliver this homework at latest on Tuesday, November 8.

1. (0.2p) Find all solutions to the equation

$$6x + 9y = 15$$

   in the ring Z_{18}.

2. (0.1p) Find all solutions to the equation

$$6x + 9y = 15$$

   in the ring Z_{19}.

3. (0.2) Find the number of solutions to an equation

$$a_1x_1 + a_2x_2 + \ldots + a_nx_n = b\ ,$$

   in a ring Z_p, where p is a prime number.

4. (0.2p) Give, and discuss, i.e., and sketch a proof of a more general result from which your answer to the previous problem follows.

5. (0.2p) Let p be a prime number. The set of all n-tuples $\bar{x} = (x_1, x_2, \ldots, x_n)$, where $x_i \in Z_p$, can be regarded as a vector space, denoted Z_p^n, with the elements $\bar{x}$ as vectors and the elements of Z_p as scalars. You do not need to verify this. However, explain why the following dotproduct

$$\bar{x} \cdot \bar{y} = x_1y_1 + x_2y_2 + \ldots + x_ny_n$$

   is not suitable for defining length of vectors, as it is done in real vector spaces.

6. (0.1p) Consider the vector space Z_p^n and the dotproduct defined as in the previous problem. To every subspace U of Z_p^n define $U^\perp$ to be the following set

$$U^\perp = \{\bar{y} \in Z_p^n \ \mid \ \bar{y} \cdot \bar{x} = 0 \text{ for all } \bar{x} \in U\}\ .$$

   Find and describe an example, i.e., find p, n and U, such that

$$U = U^\perp\ .$$