Matematiska Institutionen
KTH

Homework number 4 to SF2736, fall 2011.

Please, deliver this homework at latest on Tuesday, November 29.

1. (0.2p) Let $(G, \cdot)$ denote the group that consists of all elements in the ring Z_{20} that are invertible by multiplication. This group is isomorphic to a direct product of cyclic groups. Find this direct product of cyclic groups and describe the isomorphism.

2. (0.2p) Consider the group $\mathcal{S}_8$ consisting of all permutation of the set $\{1, 2, 3, \ldots, 8\}$. Find all possible orders of the elements of $\mathcal{S}_8$.

3. (0.3p) Show that if H and K are subgroups of an abelian group G satisfying
$$|H| \cdot |K| = |G| \qquad \text{and} \qquad |H \cap K| = 1,$$
then every element g in G can in a unique way be written as a sum
$$g = h + k,$$
of elements $h \in H$ and $k \in K$.

4. (0.3p) Show that all abelian groups of size 35 are isomorphic.