Solution to problem number 2, November 29, SF2736, fall 11.

The sides of the cube are denoted f for front, b for back, u for the upper side, d for the down, l for the left and r for the right side. We apply the lemma of Burnside, and thus we must describe the automorphism group $\mathrm{Aut}(G)$ of the cube. As the front can be moved to six positions, the side x, and then there are four rotations that keep the side x fixed, we get that

$$|\mathrm{Aut}(G)| = 24.$$

In the table below, the automorphism group is divided into six parts. The front moved to the left side will give the same type of permutations as the front moved to the right side. Thus we just present the moves to the left side. Similarly for up and down.

$\varphi \in \mathrm{Aut}(G)$	$\lvert\mathrm{Fix}(\varphi)\rvert$
id.	3^6
$(f)(b)(u\ r\ d\ l)$	3^3
$(f)(b)(u\ d)(r\ l)$	3^4
$(f)(b)(u\ l\ d\ r)$	3^3
$(f\ u\ b\ d)(r)(l)$	3^3
$(f\ u\ r)(b\ d\ l)$	3^2
$(f\ u)(b\ d)(r\ l)$	3^3
$(f\ u\ l)(b\ d\ r)$	3^2
similar	3^3
pattern	3^2
for	3^3
f to d	3^2
$(f\ l\ b\ r)(u)(d)$	3^3
$(f\ l\ d)(b\ r\ u)$	3^2
$(f\ l)(b\ r)(u\ d)$	3^3
$(f\ l\ u)(b\ r\ d)$	3^2
similar	3^3
pattern	3^2
for	3^3
f to r	3^2
$(f\ b)(u\ d)(l)(r)$	3^4
$(f\ b)(u\ l)(d\ r)$	3^3
$(f\ b)(u)(d)(l\ r)$	3^4
$(f\ b)(u\ r)(d\ l)$	3^3

Answer:

$$\frac{1}{24}(3^6 + 3 \cdot 3^4 + 12 \cdot 3^3 + 8 \cdot 3^2) = 57.$$