Problem session November 16, SF2736, fall 11.

1. In how many ways can we choose a committee in a class consisting of 11 girls and 12 boys if
   (a) the committee shall consist of 4 girls and 4 boys.
   (b) the committee shall consist of 4 girls and 4 boys, but if the boy A is chosen to the committee then the girl B cannot attend.
2. Find the number of ways we can form words of length 7 using the letters in the word DISKRET if none of the words RET, SIK or DIS may appear as subwords in the word.
3. Prove that
$$\binom{n}{r}\binom{r}{k} = \binom{n}{k}\binom{n-k}{r-k}.$$
4. Find the coefficient of x^{12} in the polynomial $(4+3x^2)^{10}$.
5. Find a formula for $S(n,2)$.
6. Show that if $\gcd(n,m)=1$ then $\phi(nm)=\phi(n)\phi(m)$
7. Find the number of positive integers d that divides the integer 129600.
8. Find the number of ways the fifteen children in a class can be placed into three rows (a row is not a queue).
9. Find the number of surjective maps f from the set $\{1,2,3,\ldots,10\}$ to $\{1,2,3,\ldots,6\}$ such that the elements $f(1)$, $f(2)$ and $f(3)$ are distinct elements.
10. Show that
$$\binom{m+n}{r} = \binom{m}{0}\binom{n}{r} + \binom{m}{1}\binom{n}{r-1} + \ldots + \binom{n}{r}\binom{n}{0}$$
11. Show that in any set of 10 people there are either four mutual friends or three mutual strangers.