Vektorformler

$$\mathbf{a}=\mathbf{i}\,a_x+\mathbf{j}\,a_y+\mathbf{k}\,a_z \qquad (0{,}01)$$

$$\mathbf{a}\cdot\mathbf{b}=a_x b_x+a_y b_y+a_z b_z \qquad (0{,}02)$$

$$\mathbf{a}\times\mathbf{b}=\mathbf{i}\,(a_y b_z-a_z b_y)+\mathbf{j}\,(a_z b_x-a_x b_z)+\mathbf{k}\,(a_x b_y-a_y b_x) \qquad (0{,}03)$$

$$\mathbf{a}\cdot(\mathbf{b}\times\mathbf{c})=\mathbf{b}\cdot(\mathbf{c}\times\mathbf{a})=\mathbf{c}\cdot(\mathbf{a}\times\mathbf{b}) \qquad (0{,}04)$$

$$\mathbf{a}\times(\mathbf{b}\times\mathbf{c})=\mathbf{b}\,(\mathbf{a}\cdot\mathbf{c})-\mathbf{c}\,(\mathbf{a}\cdot\mathbf{b}) \qquad (0{,}05)$$

$$\text{grad}\;\varphi=\mathbf{i}\frac{\partial\varphi}{\partial x}+\mathbf{j}\frac{\partial\varphi}{\partial y}+\mathbf{k}\frac{\partial\varphi}{\partial z} \qquad (0{,}06)$$

$$\text{div}\;\mathbf{a}=\frac{\partial a_x}{\partial x}+\frac{\partial a_y}{\partial y}+\frac{\partial a_z}{\partial z} \qquad (0{,}07)$$

$$\text{rot}\;\mathbf{a}=\mathbf{i}\left(\frac{\partial a_z}{\partial y}-\frac{\partial a_y}{\partial z}\right)+\mathbf{j}\left(\frac{\partial a_x}{\partial z}-\frac{\partial a_z}{\partial x}\right)+\mathbf{k}\left(\frac{\partial a_y}{\partial x}-\frac{\partial a_x}{\partial y}\right) \qquad (0{,}08)$$

$$\Delta\varphi=\frac{\partial^2\varphi}{\partial x^2}+\frac{\partial^2\varphi}{\partial y^2}+\frac{\partial^2\varphi}{\partial z^2}=(\nabla)^2\varphi=\text{div grad}\;\varphi \qquad (0{,}09)$$

symboliska vektorn del (eller nabla) $\nabla=\mathbf{i}\frac{\partial}{\partial x}+\mathbf{j}\frac{\partial}{\partial y}+\mathbf{k}\frac{\partial}{\partial z}$ (0,10)

$$\text{rot grad}\;\varphi=0 \qquad (0{,}11)$$

$$\text{div rot}\;\mathbf{a}=0 \qquad (0{,}12)$$

$$\text{rot rot}\;\mathbf{a}=\text{grad div}\;\mathbf{a}-\Delta\mathbf{a} \qquad (0{,}13)$$

$$\text{div}\;(\varphi\mathbf{a})=\varphi\;\text{div}\;\mathbf{a}+\mathbf{a}\cdot\text{grad}\;\varphi \qquad (0{,}14)$$

$$\text{rot}\;(\varphi\mathbf{a})=\varphi\;\text{rot}\;\mathbf{a}+(\text{grad}\;\varphi)\times\mathbf{a} \qquad (0{,}15)$$

$$\text{div}\;(\mathbf{a}\times\mathbf{b})=\mathbf{b}\cdot\text{rot}\;\mathbf{a}-\mathbf{a}\cdot\text{rot}\;\mathbf{b} \qquad (0{,}16)$$

$$\text{rot}\;(\mathbf{a}\times\mathbf{b})=(\mathbf{b}\cdot\nabla)\,\mathbf{a}-(\mathbf{a}\cdot\nabla)\,\mathbf{b}+\mathbf{a}\;\text{div}\;\mathbf{b}-\mathbf{b}\;\text{div}\;\mathbf{a} \qquad (0{,}17)$$

$$\text{grad}\;(\mathbf{a}\cdot\mathbf{b})=(\mathbf{a}\cdot\nabla)\,\mathbf{b}+(\mathbf{b}\cdot\nabla)\,\mathbf{a}+\mathbf{a}\times\text{rot}\;\mathbf{b}+\mathbf{b}\times\text{rot}\;\mathbf{a} \qquad (0{,}18)$$

$$\mathbf{a}\cdot(\mathbf{b}\cdot\nabla)\,\mathbf{c}=\mathbf{b}\cdot(\mathbf{a}\cdot\nabla)\,\mathbf{c}+\mathbf{b}\cdot(\mathbf{a}\times\text{rot}\;\mathbf{c}) \qquad (0{,}19)$$

$$\iiint\text{div}\;\mathbf{a}\,dv=\iint\mathbf{n}\cdot\mathbf{a}\,dS \qquad \text{(Gauss)} \qquad (0{,}20)$$

$$\iint\mathbf{n}\cdot\text{rot}\;\mathbf{a}\,dS=\oint\mathbf{a}\cdot d\mathbf{s} \qquad \text{(Stokes)} \qquad (0{,}21)$$

Teoretisk Elektroteknik/Alfvénlaboratoriet

$$\iiint \operatorname{rot} \mathfrak{a}\, dv = \iint \mathfrak{n} \times \mathfrak{a}\, dS \tag{0,22}$$

$$\iiint \operatorname{grad} \varphi\, dv = \iint \mathfrak{n} \varphi\, dS \tag{0,23}$$

$$\iint \mathfrak{n} \times \operatorname{grad} \varphi\, dS = \oint \varphi\, d\mathfrak{s}. \tag{0,24}$$

I cylinderkoordinater r, φ, z blir, om $\mathfrak{i}, \mathfrak{j}, \mathfrak{k}$ beteckna enhetsvektorerna i respektive koordinatriktningar,

$$\operatorname{div} \mathfrak{a} = \frac{1}{r} \frac{\partial}{\partial r}(r\,\mathfrak{a}_r) + \frac{\partial \mathfrak{a}_\varphi}{r \partial \varphi} + \frac{\partial \mathfrak{a}_z}{\partial z} \tag{0,25}$$

$$\operatorname{grad} V = \mathfrak{i} \frac{\partial V}{\partial r} + \mathfrak{j} \frac{\partial V}{r \partial \varphi} + \mathfrak{k} \frac{\partial V}{\partial z} \tag{0,26}$$

$$\operatorname{rot} \mathfrak{a} = \mathfrak{i} \left(\frac{\partial \mathfrak{a}_z}{r \partial \varphi} - \frac{\partial \mathfrak{a}_\varphi}{\partial z} \right) + \mathfrak{j} \left(\frac{\partial \mathfrak{a}_r}{\partial z} - \frac{\partial \mathfrak{a}_z}{\partial r} \right) + \mathfrak{k} \left(\frac{1}{r} \frac{\partial}{\partial r}(r\,\mathfrak{a}_\varphi) - \frac{\partial \mathfrak{a}_r}{r \partial \varphi} \right). \tag{0,27}$$

I sfäriska koordinater r, ϑ, φ blir

$$\operatorname{div} \mathfrak{a} = \frac{1}{r^2} \frac{\partial}{\partial r}(r^2 \mathfrak{a}_r) + \frac{1}{r \sin \vartheta} \frac{\partial}{\partial \vartheta}(\mathfrak{a}_\vartheta \sin \vartheta) + \frac{1}{r \sin \vartheta} \frac{\partial \mathfrak{a}_\varphi}{\partial \varphi} \tag{0,28}$$

$$\operatorname{grad} V = \mathfrak{i} \frac{\partial V}{\partial r} + \mathfrak{j} \frac{\partial V}{r \partial \vartheta} + \mathfrak{k} \frac{1}{r \sin \vartheta} \frac{\partial V}{\partial \varphi} \tag{0,29}$$

$$\begin{aligned} \operatorname{rot} \mathfrak{a} = {} & \mathfrak{i} \left(\frac{1}{r \sin \vartheta} \frac{\partial}{\partial \vartheta}(\mathfrak{a}_\varphi \sin \vartheta) - \frac{1}{r \sin \vartheta} \frac{\partial \mathfrak{a}_\vartheta}{\partial \varphi} \right) + \\ & + \mathfrak{j} \left(\frac{1}{r \sin \vartheta} \frac{\partial \mathfrak{a}_r}{\partial \varphi} - \frac{1}{r} \frac{\partial}{\partial r}(r\,\mathfrak{a}_\varphi) \right) + \\ & + \mathfrak{k} \left(\frac{1}{r} \frac{\partial}{\partial r}(r\,\mathfrak{a}_\vartheta) - \frac{\partial \mathfrak{a}_r}{r \partial \vartheta} \right). \end{aligned} \tag{0,30}$$

Här är ϑ polavståndsvinkeln $0 < \vartheta < \pi$ och φ longitudvinkeln $0 < \varphi < 2\pi$.

$$\operatorname{ytdiv} \mathfrak{a} = \mathfrak{n}_1 \cdot \mathfrak{a}_1 + \mathfrak{n}_2 \cdot \mathfrak{a}_2 \tag{0,31}$$

$$\operatorname{ytrot} \mathfrak{a} = \mathfrak{n}_1 \times \mathfrak{a}_1 + \mathfrak{n}_2 \times \mathfrak{a}_2. \tag{0,32}$$

Om i ortogonala koordinater x_1, x_2, x_3

$$d\mathfrak{s} = h_1\, dx_1\, \mathfrak{i} + h_2\, dx_2\, \mathfrak{j} + h_3\, dx_3\, \mathfrak{k},$$

så är

$$\operatorname{div} \mathfrak{a} = \frac{1}{h_1 h_2 h_3} \left[\frac{\partial}{\partial x_1}(h_2 h_3 \mathfrak{a}_1) + \frac{\partial}{\partial x_2}(h_3 h_1 \mathfrak{a}_2) + \frac{\partial}{\partial x_3}(h_1 h_2 \mathfrak{a}_3) \right] \tag{0,33}$$

och

$$\operatorname{rot} \mathfrak{a} = \frac{1}{h_1 h_2 h_3} \begin{vmatrix} h_1 \mathfrak{i} & h_2 \mathfrak{j} & h_3 \mathfrak{k} \\ \dfrac{\partial}{\partial x_1} & \dfrac{\partial}{\partial x_2} & \dfrac{\partial}{\partial x_3} \\ h_1 \mathfrak{a}_1 & h_2 \mathfrak{a}_2 & h_3 \mathfrak{a}_3 \end{vmatrix}. \tag{0,34}$$