Institutionen för matematik
KTH
Chaotic Dynamical Systems, SF2720, Fall 2012

Homework assignment 1

The exercises are due on September 28, 2012

(1) Let $f(x) = x^2+x$. Find all fixed points of f. Where do nonfixed points go under iteration by f?

(2) Imagine that you have a calculator with a "cos" button. You enter a number x_0, and press "cos". Then you get $x_1 = \cos(x_0)$. Continuing this process gives you a sequence $x_0, x_1, x_2, \ldots$, where $x_n = \cos(x_{n-1})$ for $n \geq 1$. Does the sequence converge to something as $n \to \infty$? If so, does the limit depend on the initial choice x_0? Prove your statements.

(3) Assume that $f : \mathbb{R} \to \mathbb{R}$ is continuous and that f has a periodic point of period 2. Show that f has a fixed point.

(4) Let $f : I \to I$ be a continuously differentiable function. Assume that p is a fixed point for f in the interior of I, and that $|f'(p)| < 1$. Show that if $g : I \to I$ is a continuously differentiable function such that

$$\sup_{x \in I} |f(x) - g(x)| < \varepsilon \quad \text{and} \quad \sup_{x \in I} |f'(x) - g'(x)| < \varepsilon,$$

where ε is sufficiently small, then the function g has a fixed point q, close to p, and $|g'(p)| < 1$. Is this necessarily true if we had assumed $|f'(p)| = 1$?

(5) Prove remark 11.1.3 (on page 301) in the text book.

(6) Do exercise 2.2.6 (on page 45).

(7) Do exercise 2.3.2 (on page 49).